NEMETH UNIFORM BRAILLE SYSTEM

CONDENSED VERSION

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Condensed Version

Compiled by Joyce H. Hull

from

The Nemeth Uniform Braille System

by

Dr. Abraham Nemeth

PREFACE
(for the condensed version)

This preface is used only in the condensed version. It is use to explain the rationale used in determining which sections to include, which ones to modify and which ones to omit. In general, the rule was dependent on how critical each section is to the understanding of the basic concepts NUBS. A significant amount of text that has been omitted is very interesting and well worth the time to read and understand. However, if it was not necessary to the understanding of the basic concepts of NUBS, it was omitted.

We cannot emphasize enough that this document IN NO WAY can serve as a replacement for the full NUBS document. It has been produced only to provide a condensed edition for those who are interested in this proposal from the perspective of determining the feasibility and utility of NUBS. It will never serve as a replacement for the original document that covers many additional elements of the System. Understanding these elements is essential to a full understanding of NUBS.

All of the section numbers are retained for easy reference to the complete document.

We have decided to include the entire Global Table of Contents from the original version. This will serve to provide a list of all the subjects that are addressed in the original NUBS document. The page numbers shown in the table of contents are from the full document and will not correspond to the page numbers in this condensed version.

Please keep an open mind and give this new System thoughtful consideration. After a period of adjustment, you will find it a joy to work with.

Joyce Hull

INTRODUCTION
(from the original full version)

The Nemeth Uniform Braille System (NUBS) is a uniform system of notation, as its name proclaims, with the capability of providing notation regardless of the field in which the notation originates, subject only to the limitations imposed by the braille medium. In this regard, NUBS emulates print in which there is no literary code, no mathematics or technical code, and no code dedicated to a particular field of knowledge.

This presentation begins with Chapter 0. Although it is not an integral part of NUBS, this chapter offers an overview of the braille system and is included for the purpose of generating interest and providing enrichment for the braille user. This is the reason for assigning 0 as the number of this chapter.

The main body of the document is organized into two major sections. The first section provides braille symbols and rules for representing print notation of the kind commonly found

in modern documents but for which the current literary code is inadequate. The following is a list of this kind of notation, all of which is discussed in the first section of NUBS:

Accent Marks	Greek Letters	Simple Fractions
Arrows	Grouping Signs	Simple Radicals
Comparison Signs	Legal Signs	Simple Superscripts and Subscripts
Currency Signs	Operation Signs	
Diacritics	Reference Signs	

A person whose interest lies primarily in the literary area can stop at the end of Literary Section which ends with Chapter 3, at which point all his needs will have been fulfilled.

The second major section of NUBS, the Scientific Section, deals with technical, primarily mathematical notation. Some items in this section are extensions of corresponding items dealt with in the first section; other items are new. Everything in the first section remains valid in the second section.

Some of the more important features of NUBS are the following:

- NUBS has been designed based on an explicitly stated set of principles and guidelines.
- NUBS has been designed with attention to the possibility of both forward and back translation by a computer.
- NUBS makes no changes in the current contraction system of Grade-2 braille.
- NUBS identifies which words are narrative and which are notational, thereby removing potential ambiguities.
- NUBS includes Teacher Alert sections at appropriate places, calling attention to the kind of information that a student must have so that he may more intelligently deal with the notation presented to him in braille.
- NUBS borrows heavily from the Nemeth Code. Therefore, the transition from one to the other should leave any user comfortable. No Nemeth Code textbook collections would become obsolete, and no intensive retraining for certification as a NUBS transcriber would be necessary.

Abraham Nemeth, Ph.D.
2008

GLOBAL INDEX

CHAPTER 1
AIMING AT A UNIFORM BRAILLE SYSTEM

1.7 Principles and Guidelines

The principles and guidelines within which NUBS will operate are essentially those set forth by BANA in its charge to the Unified Braille Code Project Committee in 1992 and with which we have agreed to comply. They are augmented by others which experience has shown to be useful and effective. Here they are:

a *NUBS will be based on the traditional 6-dot cell.* 8-dot technology is not available for writing nor for the interaction that may be required by other disciplines.

b *We will expect contracted braille (Grade 2) to be the norm in NUBS*. As currently envisioned, no contractions will be dropped and no new ones will be added. Only minor modifications of some rules may be necessary.

c *Under certain conditions, it will be desirable to use uncontracted braille for the narrative portions of the text.* When this is the case, all the other mechanisms of NUBS will still prevail. Thus, only the presence of contractions or their absence is all that is needed to distinguish between a Grade-1 and a Grade-2 document.

d *The NUBS rules will be formulated in such a way as to minimize the amount of intervention that is required by a human operator of a computer to effect correct translation from print to braille or from braille to print.*

e *Every notational symbol is either a prefix, a root, or a prefix-root construct.* A *prefix* is any one of the 7 characters in Group 7 of the Braille Alphabet. (See Section 0.7 for a discussion of how the characters of the Braille Alphabet are organized into groups.) A *root* is any braille character that contains any combination of dots 1, 2, and 3. There are 56 roots belonging to Groups 1 through 6. A prefix may be *compound*, that is, it may consist of two or more characters each of which is a prefix.

An indicator has no corresponding print symbol; but it offers important and timely information about the text with which it is associated. A prefix, simple or compound, by itself, can never correspond to a print symbol; it can only be an indicator. However, some indicators are prefixes, roots, or prefix-root combinations.

f *Within reasonable limits, a braille symbol should be identifiable without reference to the context in which it appears*.

g *NUBS will address itself to issues of format as well as to the representation of print graphics.* The format should be so designed that the reader can quickly and easily locate the information that he needs.

h *Braille symbols in NUBS will be used uniformly within a discipline from the most elementary to the most advanced level, and across all disciplines within the scope of*

NUBS. The braille user will learn as much or as little of NUBS as he needs for his current activities.

i *Braille symbols will be designed without regard to their meaning.* The dollar sign is often encountered in a context that has nothing to do with dollars. The term "dollar sign" simply reflects the most common use of that symbol.

j *From time to time it will be necessary to insert a transcriber's note into the text. A mechanism must exist by which the reader can distinguish between the words of the transcriber and those of the author*.

k *NUBS symbols will be so devised that they will be easily identified by the use of mnemonic prompts*. Because a robust NUBS requires a large number of symbols, they would be difficult to recognize or to recall without substantial mnemonic assistance. Several mechanisms are available to provide such assistance. When graphic symbols are symmetric in print, they will also be symmetric in braille. Families of related symbols will share either a common prefix or a common root. (See Section 1.7e, above). As appropriate, the geometry of a dot pattern in braille will approximate the geometry of the corresponding symbol in print.

EXAMPLES (of mnemonics)

Mnemonic by Symmetry

⠐⠣	left parenthesis	⠐⠜	right parenthesis
⠨⠣	left bracket	⠨⠜	right bracket
⠨⠪	less than	⠨⠕	greater than
⠸⠌	slash	⠸⠡	backslash

Mnemonic by Common Root (especially valuable for emphasized phrases)

⠸⠸⠇	begin boldface phrase	⠸⠸⠱	end boldface phrase
⠸⠸⠊	begin italic phrase	⠸⠸⠻	end italic phrase
⠸⠸⠉	begin underlined phrase	⠸⠸⠤	end underlined phrase
⠸⠸⠠	begin upper-case phrase	⠸⠸⠄	end upper-case phrase

Mnemonic by Common Prefix

⠨	prefix for all lower-case Greek letters
⠸	prefix for all upper-case Greek letters
⠈	prefix for all currency symbols

Mnemonic by Geometric Approximation

⠨⠪	left arrow	⠨⠕	up arrow
⠨⠪	right arrow	⠨⠕	down arrow

l *NUBS will be extendable in a systematic manner*. New symbols will not conflict with existing ones, and they will be subject to the same rules as the existing ones in similar situations. The prefix-root mechanism, discussed in Section 1.7e, is the means by which this System can be extended with no theoretical limit. This mechanism will also make it

particularly easy to parse a braille text into its component symbols and indicators. This is the first step in performing a back-translation from braille to print.

m *NUBS will preserve commonly accepted print practices with respect to capitalization, punctuation, spelling, grammar, or general orthography in the English language.* We aim to convey an accurate picture of the printed text. We will not substitute non-standard for standard abbreviations or punctuation marks. A test of the accuracy of the transcription is to perform a reverse transcription and compare the result, aside from format, with the original.

n *The just-in-time principle requires that the user have adequate knowledge of the notation with which he is dealing precisely at the point that he needs such information.* Thus, fraction indicators tell the user precisely when a fraction begins and ends, as well as the order of complexity of the fraction. If radicals are nested, the user is informed just as the radical begins as to the order of its complexity.

o *The minimal enclosure principle requires the avoidance to the greatest extent possible of real or of phantom enclosures in braille when none exist in print.*

p *The preserve-structure principle requires that if two notational expressions in print have the same structure but differ only in content, the same should be true in the braille representation.*

EXAMPLE (of preservation of structure)

Expressions Exhibiting the Same Structure but Differing in Content

  $\frac{1}{3}h$ $\frac{1}{c}h$

q *The equal-access principle requires that infrequently occurring structures should not be ignored.* What is an infrequent occurrence to one user is an everyday occurrence to another.

r *Space saving will not be used as a reason for overriding any of the foregoing principles.*

s *The presence or the absence of a space next to a symbol will not be used as a means of identifying a symbol.*

t *As NUBS develops, new and useful principles may emerge. If they do, they should be added to the above list and they should become as binding as their predecessors.*

CHAPTER 2
NARRATIVE AND NOTATIONAL TEXT: LIVING TOGETHER

2.0 Chapter 2 Reference List

Description	Symbol

Comparison Signs in This Chapter

equals
less than

Currency Signs in This Chapter

dollar sign

Indicators in This Chapter

base level
begin/end simulated braille
begin italicized phrase
begin notational phrase that starts with a digit
begin notational phrase that starts with a non-digit
begin upper-case phrase
end italicized phrase
end notational phrase
end upper-case phrase
italics, one word
notational
numeric
punctuation
subscript
superscript
toggle between narrative and notational components of a hybrid word
upper case, one letter
upper case, one word

Numeric Signs and Notational Punctuation

zero		six	
one		seven	
two		eight	
three		nine	
four		comma	
five		decimal point	

Operation Signs in This Chapter

plus
slash

Punctuation Marks in This Chapter

apostrophe
colon
comma
narrative
parentheses
left
right
period

notational		narrative	
dash		notational	
double quotes		question mark	
left oriented		semicolon	
right oriented		slash	
hyphen		underscore	

Radicals in This Chapter

radical sign	
termination sign	

2.1 The Duality Problem

Since we have committed ourselves to respecting BANA mandates with regard to the development of NUBS, we will expect contracted braille to be the norm for users of this System. A special requirement may, from time to time, dictate the use of uncontracted braille. When this occurs, all the mechanisms for distinguishing between narrative and notational text must still be implemented.

We must devise a method to indicate which meaning (narrative or notational) of a braille symbol is intended, and for a reader to determine how to interpret the meaning of any braille symbol. That is, we need a way for two meanings to share the same braille symbol. We will offer a rigorous definition of a ***word*** in Section 2.4. Until then, our intuitive understanding will have to serve.

2.2 Word: The Basic Building Block

Words come in many different guises. Our first impulse is to think of a word as in a novel, delimited at both ends by a space. This simplistic approach turns out, upon deeper analysis, to cause trouble. Is "self-control" one word or two? What about "U-boat," "3-dimensional," "never-to-be-forgotten," or "what-you-see-is-what-you-get"? What about abbreviations, acronyms, mnemonics—are they words? Is a single letter, like *x*, a word? We need to give firm answers to such questions in order to make this System work without ambiguity.
Here is a preliminary intuitive approach as to what is a word. A word may be an acronym, a mnemonic, or an abbreviation. A word may be a string of symbols that constitutes a notational unit such as the formula for a chemical compound, a mathematical equation, or others that may emerge as we proceed. In particular, a word may be a single letter, a single digit, or a single symbol if these are surrounded by delimiters. A string of digits, with or without commas, decimal points, or both, surrounded by delimiters, is a word.

Now, we need to refine this preliminary, intuitive concept so that it becomes unambiguous and workable. This will lead us to the solution of the duality problem that we raised above. Accordingly, we define a ***delimiter*** in Section 2.3 and a ***word*** in Section 2.4. Hopefully, the terms we define and use will evoke the same understanding among those who work with and study this System provided that we present and define them with sufficient care.

2.3 Delimiter

We designate the following as *delimiters:*

a the space
b the hyphen
c the slash
d the dash

In a hard-copy document, the transition to a new line or to a new page is also a delimiter. In an electronic document, the carriage return, the carriage-return/line-feed combination, and the form-feed are also delimiters. Even if not physically present, there is, by agreement and for technical reasons, a phantom delimiter at the beginning and at the end of every document. The concept of a delimiter applies to a braille, print, or electronic document.

Of the above named delimiters, the space, the hyphen, and the slash are conditional; that is, they cease to be delimiters under specific circumstances, as follows:

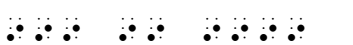
The space is not a delimiter when it occurs:

a in a long string of digits to partition that string into shorter substrings of equal length to improve readability, or to preserve the format of a string of numbers.
b on either side of a comparison sign or on either side of an operation sign.
c between a quantity and its associated abbreviation of measure. The quantity may be a number, a letter, or an expression that represents a number. However, even though the number and its abbreviation constitute a single notational word, the notational indicator must nevertheless precede the abbreviation if the abbreviation is a single English letter, lower- or upper-case, or if the abbreviation is a string of letters which constitutes a short-form word.

EXAMPLES (regarding the space as a non-delimiter)

1 
1 000 000 000
(A notational word; the spaces within a long number are not delimiters.)

2 
π = 3.14159 26535 89793 …
(One notational word; the ellipsis is in place of another group of digits.)

3 
999 99 9999
(One notational work; preserves the format of a social security number.)

4
1 234 345 4567

(One notational word; preserves the format of a telephone number.)

5

30 m, or 300 cm

(The abbreviation for meters is a single letter; the abbreviation cm is not a short-form word.)

The hyphen and the slash are not delimiters when the components on either side of them are in the same mode.

EXAMPLES (Hyphens and slashes are not delimiters.)

6 self-control

7 what-you-see-is-what-you-get

8 men aged 65-75

9 (a+b)/2 (A slash between two notational components)

10

Archimedes (287-212 B.C.)

(The space before B.C. is a delimiter since B.C. is not an abbreviation of measure.)

11 10/12/1492

12 999-99-9999 (Format for a social security number)

13 555-1212 (A telephone number)

14 I/O (input/output)

In each of Examples 6-14, the components (narrative or notational) on both sides of a hyphen or a slash are of the same mode.

When the component to the left of a hyphen or a slash is narrative and the component to the right of that hyphen or slash is notational, the notational component must be preceded by a notational indicator if it does not begin with a digit, and by a numeric indicator if it does.

EXAMPLES (of constructs containing a hyphen or a slash in which the left component is narrative and the right component is notational)

15 Bar-x (Since the second component is notational and is not a digit, a notational indicator is required.)

16 uranium-235 (Since the second component in notational and begins with a digit, a numeric indicator is required.)

If the component to the left is notational and the component to the right is narrative, we have a hybrid construct and the hyphen or the slash must be preceded by dot 5. Contractions should be used in the narrative component, if possible.

EXAMPLES (of hybrid constructs containing a hyphen or a slash where the left component is notational and the right component is narrative)

17 ⠌⠤⠙⠊⠍⠑⠝⠎⠊⠕⠝⠁⠇ 3-dimensional

18 ⠌⠭⠤⠊⠝⠞⠑⠗⠉⠑⠏⠞ x-intercept

19 ⠠⠁ ⠌⠊⠝⠉⠓⠤⠗⠥⠇⠑⠗ A 6-inch ruler

20 ⠌⠁⠤⠞⠕⠤⠁ 1-to-1

21 ⠌⠝⠤⠋⠕⠇⠙ ⠌⠝⠤⠞⠥⠏⠇⠑ ⠌⠝⠤⠜⠗⠽ n-fold, n-tuple, n-ary

22 ⠌⠭⠤⠕⠝⠸⠌⠭⠤⠕⠋⠋ X-On/X-Off

In each of Examples 17-22, the component to the left of the hyphen or slash is notational and the component the right of the hyphen or slash is narrative. Contractions are used in the narrative components when possible.

The dash is an unconditional delimiter.
With respect to runovers, the dash and the slash behave like hyphens: that is, they may be used between components at the end of a line if they, by chance, fall there. Unlike the hyphen, however, if the dash or the slash cannot be accommodated at the end of a braille line, they may be placed at the beginning of the next line.

2.4 Words: An Analysis

A *word* is a string of symbols between two delimiters but which has no interior delimiters. Accordingly, the delimiters which define a word are not part of that word. The delimiters which define a word need not be alike.

A word may be *narrative, notational,* or *hybrid.*

A word is *narrative* if it is:

a a dictionary word with its usual meaning
b an abbreviation that does not contain any of the notational features listed below
c an acronym
d a plural, possessive, or ordinal ending
e a part-word ending that converts words into adjectives or nouns like "fold," "tuple," "ary"
f neither notational nor hybrid (See below.)

EXAMPLES (of narrative words)

1 dissolve (a dictionary word)

2 Mr. (an abbreviation)

3 e.g. (an abbreviation)

4 wysiwyg (an acronym)

5 brl─zylx (neither notational nor hybrid)

A word is *notational* if it:

a is or contains an operation sign or a comparison sign.
(These are listed in the Basic Literary Symbol Set and in the Mathematics Symbol Set.)
b is or contains a fraction or a radical.
(Fractions and radicals are discussed in Chapters 3 and 5.)
c contains subscripts, superscripts, modifiers, overscripts or underscripts.
(These are discussed in Chapters 3 and 5.)
d contains digits. (Digits are listed in Section 3.0.13 under Numeric Signs, in the Basic Literary Symbol Set.)
e is or contains single letters that are not English words.
f is or contains a letter from a non-English alphabet, an accented letter or a diacritic or a special letter. (Other Alphabets are discussed in Section 3.9. Accent marks and diacritics are discussed in Sections 3.4.1 and 3.4.2)
g contains symbols that are not part of the Basic Literary Symbol Set. (All the symbols of this symbol set are listed in the Basic Literary Symbol Set.)
h is any of the above with attached punctuation that is not a delimiter.

EXAMPLES (of notational words)

6 $2.98 (Contains digits)

7 H_2O (Contains a subscript)

8 $E = mc^2$ (Contains a comparison sign, a single letter that is not a word, and a superscript)

9 x/y (Contains letters that are not words)

A word is ***hybrid*** if it contains at least one narrative component and at least one notational component. The components may be in any order. In hybrid words, when proceeding from one component to the next, either notational to narrative or narrative to notational, use dot 5 as a toggle between the components. Contractions should be used in narrative components when possible.

EXAMPLES (of hybrid words)

10 ⠼⠉⠗⠙ 3^{rd} (The 3 is notational; the "rd" is its narrative ordinal ending.)

11 ⠼⠁⠌⠎⠞ 1^{st} (The 1 is notational; "st," which may be contracted, is its narrative ordinal ending.)

12 ⠰⠭⠰⠊⠎ x_is (The plural of x_i . The s is in the host font. The first dot returns us to the base level; the second dot 5 asserts that the s is a narrative plural ending in the host font, and not a variable multiplier of *x* sub *i*.)

13 ⠼⠃⠌⠝⠙ ⠼⠉⠗⠙ ⠼⠙⠌⠹⠙ ⠼⠑⠌⠹⠑ 2^{nd}, 3^{rd}, 4^{th}, 5^{th}
(These are hybrid words with narrative ordinal endings. The narrative comma is used after the narrative ordinal ending.)

14 ⠰⠊⠌⠗⠕⠺ ⠯ ⠰⠐⠣⠊⠐⠬⠼⠁⠣⠌⠝⠙ ⠉⠕⠇⠥⠍⠝ i^{th} row and $(j+1)^{th}$ column
(Ordinal endings may be attached to expressions other than digits.)

RULES (concerning words)

a *The rules below apply only to words that are not part of a phrase or passage.* (to be discussed below).

b *A narrative word requires no special treatment; it is simply brailled or read as contracted braille text.*

c *If a notational word begins with a digit (a symbol which represents 0 through 9,) that word must begin with the numeric indicator* ⠼. All the symbols in such a word are written and read as notational.

d *If a notational word begins with a non-digit, its first character in braille must be in the notational indicator* ⠰ . The notational indicator is a direct descendant of the letter sign of the literary code. Just as the letter sign of the literary code does not permit the letter to be interpreted as an alphabetic whole-word contraction, so the notational indicator of NUBS does not permit the characters in a notational word to be interpreted as Grade-2 contractions.

e *If a word is hybrid, its components must be separated by inserting dot 5 between them.* If the first component is narrative, Rule b applies to it. If the first component is notational, Rule c or d applies to it. The status of subsequent components alternates between narrative and notational, the first component setting the rhythm. The beginning of each component except the first is delimited by dot 5, and its status (narrative or notational) is known because of this alternation. Therefore, no further status indication is necessary.

2.5 Phrases and Passages: An Analysis

A phrase and a passage are so closely related that we will discuss them together.

A phrase or a passage consists of a string of three or more consecutive related words. Adjacent words in a phrase or passage may share the delimiter between them. The interior delimiters in a phrase or passage are part of that phrase or passage, although they are not part of the component words.

We think of a ***phrase*** as a short string of words embedded within the host text, with no change in format as we enter and then leave the phrase.

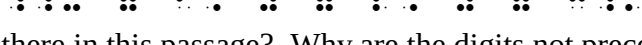
We think of a ***passage*** as a longer string of words which is displayed rather than embedded. The display may be either spatial or non-spatial. Thus, a passage is generally recognized by:

- a change in the host format
- the host text is interrupted
- in print, the passage may be separated from the preceding and following host text by blank lines
- the passage itself may be indented relative to the host margin
- it may be centered

A phrase or passage is ***narrative*** when it is displayed and contains only narrative words; otherwise, we just have a string of narrative words.

A phrase or passage is ***notational*** if at least one word therein is notational.

EXAMPLES (of notational phrases)

1 x = 1, y = 2, z = 3 
(How many words are there in this passage? Why are the digits not preceded by the numeric indicator?)

2 (x = 1, y = 2, z = 3) 
(Why is there just one notational indicator at the beginning? Which rule permits this?)

3 This is a 1-to-1 mapping.
(1-to-1 1is a hybrid phrase)

A phrase or passage is a ***hybrid construct*** if its components are joined by either hyphens or slashes, and if some components are narrative and some are notational.

RULES (concerning phrases and passages)

a *A phrase or passage that is narrative needs no indication.*

b *A passage that is displayed needs no indication.* If any part of the display is notational, the whole passage is notational, but narrative components, if any, must be rendered in uncontracted braille. If the displayed passage contains no notational components, then the passage is narrative and should be rendered in contracted braille.

c *A phrase that is notational and is enclosed within grouping signs of any kind, is treated as a single notational word.* Only the notational indicator ⠸ is required before the beginning enclosure symbol.

d *A notational phrase that is not enclosed, and whose first word begins with a digit, requires the digital notational indicator* ⠸⠼ *as its first symbol.*

e *A notational phrase that is not enclosed, and whose first word begins with a non-digit, requires the double notational indicator* ⠸⠸ *as its first symbol.*

f *Every word in a notational phrase that begins with a digit, except the first word, must be preceded by the numeric indicator.*

g *A notational phrase that is not enclosed must be terminated by the terminal notational indicator* ⠸⠄ . If a notational phrase ends in the middle of a word, the same terminal notational indicator must be used.

h *If a notational phrase is enclosed, no terminal notational indicator is needed; the closing enclosure symbol is sufficient.*

One of the goals of this chapter is to make the numeric indicator and the various notational indicators as unobtrusive as possible without compromising the requirement for unambiguous transcribing and reading.

CHAPTER 3
THE TRUNK OF THE TREE

CHAPTER 3 TOPICS

Many topics are addressed in this chapter. They are listed in the Global Table of Contents on pages *iv* through *vi* and are not listed here.

The purpose of this chapter is to make the literary code more robust. If you look at the subsections in the Basic Literary Symbol Set or at the list of topics which introduces this chapter, you will realize that there are many areas in which the current literary code is incomplete.

3.0 Basic Literary Symbol Set

This symbol set contains signs likely to be found in everyday printed materials, and constitutes the Basic Literary Symbol Set. Signs that are less common have been deferred to Chapter 5, which deals with material that is more technical. The signs in this symbol set have been sorted into categories and the categories have been alphabetized.

3.0.1 ACCENT MARKS AND DIACRITICS

acute		ligature	
bar		macron	
breve		primary stress mark	
cedilla		ring	
circumflex		schwa	
diaeresis		secondary stress mark	
grave		tilde	
hat		umlaut	
hooked n			

3.0.2 ARROWS

east		south	
north		west	
north-south		west-east	

3.0.3 COMPARISON SIGNS

equals ⠶
greater than ⠐⠕
less than ⠐⠣

3.0.4 CURRENCY SIGNS

cent	⠠⠉	franc	⠠⠋
dollar	⠠⠙	pound sterling	⠠⠇
euro	⠠⠑	yen	⠠⠽

3.0.5 ENGLISH ALPHABET, LOWER CASE

a	⠁	n	⠝
b	⠃	o	⠕
c	⠉	p	⠏
d	⠙	q	⠟
e	⠑	r	⠗
f	⠋	s	⠎
g	⠛	t	⠞
h	⠓	u	⠥
i	⠊	v	⠧
j	⠚	w	⠺
k	⠅	x	⠭
l	⠇	y	⠽
m	⠍	z	⠵

3.0.6 ENGLISH ALPHABET, UPPER CASE

This list is the same as 3.0.5 except that each letter is preceded by dot 6.

3.0.7 GREEK ALPHABET, LOWER CASE

alpha	⠨⠁	nu	⠨⠝
beta	⠨⠃	xi	⠨⠭
gamma	⠨⠛	omicron	⠨⠕
delta	⠨⠙	pi	⠨⠏
epsilon	⠨⠑	rho	⠨⠗
eta	⠨⠱	sigma	⠨⠎
zeta	⠨⠵	tau	⠨⠞
theta	⠨⠹	upsilon	⠨⠥
iota	⠨⠊	phi	⠨⠋
kappa	⠨⠅	chi	⠨⠯
lambda	⠨⠇	psi	⠨⠽
mu	⠨⠍	omega	⠨⠺

3.0.8 GREEK ALPHABET, UPPER CASE

This list is the same as 3.0.7 except that the prefix before letter is changes from ⠨ to ⠸ .

3.0.9 GROUPING SIGNS

angle brackets
left ⠐⠣ right ⠐⠜
braces
left ⠐⠪ right ⠐⠻
brackets
left ⠐⠣ right ⠐⠜

double quotes
left oriented ⠠⠶ right oriented ⠠⠶ unoriented ⠠⠶
parentheses
left ⠶ right ⠶
single quotes
left oriented ⠐⠦ right oriented ⠴ unoriented ⠄
transcriber's note enclosure
left ⠠⠣ right ⠠⠜

3.0.10 INDICATORS

base level ⠰
begin/end simbraille ⠰⠶
begin-fraction ⠆
boldface
begin phrase ⠘⠆⠆
end phrase ⠘⠲
one character ⠘⠆
one word ⠘⠃
end fraction ⠰
end notation ⠸⠲
foreign language
begin phrase ⠨⠆⠆
end phrase ⠨⠲
one character ⠨⠆
one word ⠨⠃
italics
begin phrase ⠈⠆⠆
end phrase ⠈⠲
one character ⠈⠆
one word ⠈⠃
ligature ⠐⠶
numeric ⠼
punctuation ⠲
reference ⠐⠜
runover ⠳
subscript ⠢
superscript ⠔
termination ⠰
transcriber's enclosure
left ⠠⠣
right ⠠⠜
underlined
begin phrase ⠸⠆⠆
end phrase ⠸⠲
one character ⠸⠆
one word ⠸⠃
upper case
begin phrase ⠠⠠⠠
end phrase ⠠⠠
one letter ⠠
one word ⠠⠠

3.0.11 LEGAL SIGNS

copyright ⠐⠉
credit ⠐⠒
prescription ⠐⠗
registered ⠐⠗
trademark ⠐⠞

3.0.12 MISCELLANEOUS SIGNS

ampersand ⠯
degrees ⠴

at
bullet
caret
crosshatch
infinity
percent
per mil
vertical bar

3.0.13 NUMERIC SIGNS

zero
one
two
three
four
five
six
seven
eight
nine
comma
decimal point

3.0.14 OPERATION SIGNS

asterisk
divided by
minus
plus
slash
times cross
times dot

3.0.15 PUNCTUATION

angle brackets
  left
  right
apostrophe
asterisk
backslash
braces
  left
  right
brackets
  left
  right
colon
comma
  narrative
  notational
dash
  long
  short
ditto marks
double quotes
  left oriented
  right oriented
  unoriented
ellipsis
exclamation mark
feet, minutes
hyphen
inches, seconds
inches, seconds
parentheses
  left
  right
period
  narrative
  notational

question mark
semicolon
single quotes
left-oriented right-oriented unoriented
slash
underscore

3.0.16 REFERENCE SIGNS

asterisk
dagger
double dagger
paragraph sign
pointing finger
reference indicator
section sign

3.0.17 SIMPLE FRACTIONS

begin-fraction indicator
end-fraction indicator
linear fraction bar
one-fourth as a single graphic
one-half as a single graphic

3.0.18 SIMPLE RADICALS

radical sign
with vinculum without vinculum
termination indicator

3.0.19 SIMPLE SUBSCRIPTS AND SUPERSCRIPTS

baseline level indicator
subscript indicator
superscript indicator

3.1 The English Alphabet

3.1.1 Lower-Case Letters

The braille characters we choose to represent the letters of the English alphabet are those originally chosen by Louis Braille, and which are standard all over the world.

3.1.2 Upper-Case Letters

Upper-case letters are also part of the English Alphabet. In our 6-dot Braille System, we cannot afford to set aside an additional set of 26 characters for representing the upper-case

letters. Therefore, we devise an indicator which asserts that the next letter is upper case. This symbol is called the ***upper-case letter indicator.*** Its representation is:

upper-case letter indicator 

When this indicator precedes a contraction or a short-form word, only the first letter of that contraction or short-form word is indicated as upper case.

3.1.3 Upper-Case Words

If a word contains letters all of which are upper case, we indicate this by using the ***upper-case word indicator*** whose representation is:

upper-case word indicator 

The indicator is placed before the first letter in the sequence of upper-case letters. The delimiter which marks the end of the word then also terminates the effect of the indicator. If capitalization begins in the interior of a word, we simply use the upper-case word indicator at the point at which capitalization begins. If capitalization terminates in the interior of a word, we recognize this with the ***terminate upper-case indicator*** whose representation is:

terminate upper-case indicator 

Please note that this indicator is the same as the termination sign of the literary code which serves the same purpose.

3.1.4 Upper-Case Phrases and Passages

If a phrase or passage is capitalized, as is often the case with centered headings, we indicate this by using the ***begin upper-case phrase indicator*** whose representation is:

begin upper-case phrase indicator 

to introduce the capitalized phrase or passage, and the ***end upper-case phrase indicator*** whose representation is:

end upper-case phrase indicator 

to terminate it.

We now add the four indicators to our growing system..

upper-case letter indicator
upper-case word indicator
begin upper-case phrase indicator
end upper-case phrase indicator

3.2 Punctuation

Any attempt to create a Uniform Braille System must address the punctuation of the literary code with a view to restructuring it. Punctuation marks will be considered one by one.

3.2.1 Punctuation Indicator

We first introduce the punctuation indicator with only a brief statement; its use will be elaborated as we proceed. The *punctuation indicator* asserts that the symbol that follows is a punctuation mark rather than a digit or a contraction. Its representation is:

punctuation indicator ⠸

3.2.2 Parentheses

The current code provides no distinction between opening and closing parentheses. In technical environments, this distinction is crucial. The following symbols for parentheses are proposed:

parentheses: left ⠷ right ⠾

In a Grade-2 environment, these symbols are the contractions for "of " and "with." Therefore, they require the punctuation indicator ⠸ to assert that they are punctuation marks. In a notational environment where there are no contractions, these one-cell symbols require no further modification. In a Grade-1 transcription, the punctuation indicator would still be required before a parenthesis that is in contact with a narrative word (see Section 1.7c). When preceded by the punctuation indicator, these symbols lose their meaning as contractions and acquire the meaning as punctuation marks. One usually thinks of the punctuation indicator as being used in a notational context to distinguish punctuation marks from digits; the use of the punctuation indicator in a narrative context vs. its non-use in a notational context in the case of parentheses is therefore somewhat of a switch.

3.2.3 Brackets, Braces, Angle Brackets

In English-speaking countries outside of the United States, brackets are sometimes called square brackets. The official literary code provides for brackets. However, the right bracket is an ambiguous construct; it can be interpreted as a parenthesis followed by an apostrophe. In addition, it is not a prefix-root construct, in violation of one of our principles. The following symbols are proposed for brackets:

brackets : left ⠸⠷ right ⠸⠾

These symbols should be used whether they occur in a notational context or a narrative context.
The following symbols are proposed for braces:

braces: left ⠸⠷ right ⠸⠾

These symbols should be used whether they occur in a notational context or a narrative context. They do not conflict with any Grade-2 contractions and thus do not require the punctuation indicator.

In print, the most common use for angle brackets is for enclosing e-mail addresses or for enclosing keystrokes. The following symbols are proposed for angle brackets:

angle brackets: left ⠈⠣ right ⠈⠜

In print, the angle brackets are identical to the signs for "less than" and "greater than. Accordingly, they will also be identical in NUBS. This is in compliance with the principle that requires the transcription of notation regardless of meaning. These symbols do not require a punctuation indicator.

In all the grouping symbols proposed so far–parentheses, brackets, braces, and angle brackets–the left and the right components are left-to-right symmetric in NUBS as they are in print.

3.2.4 Double Quotes

We will now consider the issue of double quotes which is probably the hairiest issue in the punctuation system.

First, we will agree to use oriented quotes in braille when there are oriented quotes in print, and to use unoriented quotes in braille when there are unoriented quotes in print. To do otherwise would deprive the user of knowing how quotes are used in the mainstream world of print. Weighing all considerations of potential conflict, we make the following proposal:

double quotes
left-oriented ⠨⠦ right-oriented ⠨⠴ unoriented ⠨⠶

Finally, the representation for unoriented quotes can also be used to represent "inches" or "seconds" as well as ditto marks. If it is clear that two apostrophes have been used in print for the representation of "inches" or "seconds" instead of the double quote, two apostrophes may also be used in braille for this purpose. The same is true for the representation of ditto marks.

3.2.5 Single Quotes

Like double quotes, single quotes may be either oriented or unoriented. The same techniques in print are used to achieve this orientation. That is, an accent mark is used for a left-oriented single quote, and an apostrophe is used for a right-oriented single quote. Thus, we make the following proposals:

apostrophe, prime ⠄
feet, minutes ⠄
single quote
left-oriented ⠈⠠ right-oriented ⠨⠠ unoriented ⠄

Note the geometric similarity between the braille and the print representations of oriented single quotes.

3.2.6 Slash, Backslash, Asterisk

The official literary code does not, with justification, provide for the backslash. However, we want our system to be sufficiently robust to contain all the ASCII characters, and so we will make provision for the backslash.

The asterisk also presents a problem.. Other than by context, the reader cannot tell whether the word ⠐⠑⠍⠊⠝⠊⠝⠑ "feminine" contains an asterisk or consecutive "in" contractions. We propose the following assignments for the three punctuation marks discussed above:

slash ⠸⠌ backslash ⠸⠡ asterisk ⠸⠔

Note the geometric symmetry between the slash and the backslash in braille in imitation of the corresponding symmetry between these same symbols in print. Note also that in each of the above symbols, the ⠸ prefix is part of the symbol and is not the punctuation indicator. They are not Grade-2 contractions and do not require the punctuation indicator in any context.

3.2.7 Ellipsis

In print, the ellipsis is represented by three equally-spaced dots. It is commonly thought that these dots are periods. In an ASCII file or on a typewriter, the ellipsis is approximated by three periods. In most word processors, an ellipsis is often represented by three closely-spaced dots, so as to occupy the space of one print character.

The official literary code represents a horizontal ellipsis by three consecutive dots 3 which, although intended to represent dots, can only be interpreted by the rules of the literary code as three apostrophes.

We propose the following to represent ellipsis:

horizontal ellipsis ⠄⠄⠄

Note this representation conforms to the prefix-root structure required by NUBS.

3.2.8 Semicolon, Colon, Exclamation Mark, Question Mark

These punctuation marks are frequently thought of as "final" punctuation marks, meaning that they occur at the ends of clauses or sentences. In fact, these punctuation marks, with increasing frequency, are found in the middle of words, both in a narrative and a notational context. And, for expository purposes, they sometimes stand alone.

In everyday literature, the colon is, besides its normal function, used to separate the hours from the minutes in telling the time of day. The colon is often used to separate major parts of a sentence.

In mathematics, the exclamation mark is used as the factorial symbol.

The following proposals for the above four punctuation marks agree with how they are represented in the official literary code:

semicolon	⠰⠆	exclamation mark	⠰⠖
colon	⠰⠒	question mark	⠰⠦

In a narrative Grade-2 context, these punctuation marks will be represented as shown above; but in a notational context they will require the punctuation indicator.

3.2.9 Hyphen, Dash, Underscore

The hyphen poses no problem and will be represented as in the official literary code. For the sake of completeness, it will be listed below even though there is no change.

In general, since the hyphen cannot be confused with a digit, it does not require a punctuation indicator. The only time a punctuation indicator might be required is to distinguish a hyphen from the initial "com" contraction in a Grade-2 environment; but this situation is extremely rare. However, braille users are not exempt from having to deal with rare situations, and so the mechanism exists for making the distinction if necessary.

In print, there is no discernible distinction between a hyphen and a minus sign. This will also be true in NUBS.

In print, the dash is represented by a continuous horizontal line. In the official literary code, the dash is approximated by two hyphens. We also use this "approximate" dash in an ASCII file or on a typewriter where there is no true dash. But two hyphens are not a true dash and should not be used to represent a dash. Furthermore, the official literary code distinguishes between a short dash, represented by two hyphens, and a long dash, represented by four hyphens. In print, there are no symbols corresponding to the short dash or the long dash of braille. In print, a popular mark-up and typesetting language, (TeX), provides for an em-dash, the width of the letter *m*, and an en-dash, the width of the letter *n*. In addition, typesetters make the dash have whatever length suits their needs. In print, the various lengths of the dash are used primarily to enhance the appearance of the text and should not be replicated in braille. The dash, unlike the hyphen and the space, is an unconditional delimiter.

The underscore is a punctuation symbol which has no representation in the official literary code. Here we show its representation only when it stands alone, that is, when it underlines a space. Underlining, which is implemented for larger segments of text, will be dealt with in Section 3.6 together with other fonts.

In view of the foregoing discussions, we make the following proposals:

hyphen	⠤
dash	

long ⠤⠤⠤ short ⠤⠤
underscore ⠤⠤

The hyphen, the dash, and the underscore do not require a punctuation indicator in any context, except in the rare case of the hyphen when it could be taken as the "com" contraction.

3.2.10 Period, Comma

Although these are the two most common punctuation marks, we have left them for last because they are probably more controversial than any of the other punctuation marks.

In a Uniform Braille System, the decimal point should have the same representation as the period. We propose the following for the period:

period
narrative ⠲ notational ⠲

With the implementation of this proposal, we will not have a 100% pure uniform system, because we will then have two representations for the period. However, any proposal will have to make some compromises as long as we are committed to retaining Grade 2 in its present form with minimum change. I believe that this proposal will cause a minimum amount of disruption to the goal of a 100% pure uniform system.

Therefore, we make the following proposal for the comma:

comma
narrative ⠂ notational ⠂

With these proposals in place, the notational period will no longer require a punctuation indicator. Thus, the only punctuation marks that will require a punctuation indicator are:

In notational text
semicolon
colon
exclamation point
question mark
In narrative text
parentheses

Without collecting statistics, I submit that, collectively, these punctuation marks do not constitute a large body of text, so that the punctuation indicator is mostly unobtrusive.

3.3 Numbers

At last we must come to grips with the issue of numbers. This issue has turned out to be the most controversial issue in the development of any uniform system. The full document

defining NUBS takes a tour of three competing number systems, pointing out the features, both positive and negative, of each. This leads to the conclusion that the ***dropped number system*** holds out the greatest promise for the development of a uniform braille system, and thereafter we shall proceed along that path.

3.3.4 The Numeric Indicator

We have already shown that the numeric indicator must introduce a notational word that begins with a digit. Elsewhere we stated that the numeric indicator was more historical than functional. At this point, we undertake a further analysis of the numeric indicator. The symbol for the numeric indicator is:

numeric indicator ⠼

In NUBS, the numeric indicator serves two functions: First, it serves as a dot locator for the braille character that follows it, so that the braille reader can easily ascertain that the next character is in the lower part of the cell. Consider the example:

He made a 1 under par in golf.

⠓⠑ ⠍⠁⠙⠑ ⠁ ⠼⠁ ⠥⠝⠙⠑⠗ ⠏⠁⠗ ⠊⠝ ⠛⠕⠇⠋⠲

Imagine what the result would be if the numeric indicator were omitted! The character for the 1 has no right-hand dots, and the character following the 1 has no left-hand dots. This leaves the 1 relatively isolated. Only the most experienced braille users would locate the single dot representing the 1 easily and correctly. The numeric indicator orients this single dot immediately and precisely.

The second function of the numeric indicator is to distinguish a digit from a lower-sign whole-word contraction. Consider the example:

There were 7 birds on the fence. ⠮⠻⠼ ⠻ ⠼⠻ ⠃⠊⠗⠙⠎⠼ ⠕⠝ ⠼ ⠋⠑⠝⠉⠑⠲

The first ⠻ is the contraction for "were;" the next ⠻ is preceded by the numeric indicator which asserts that this is the digit 7 and not the lower-sign whole-word contraction for "were."

These are the only two functions of the numeric indicator.

3.3.5 Comma and Decimal Point in Numbers

A comma in a number is represented just like a comma in any other notational word. Longer numbers may contain several commas to separate the digits into equal-sized groups (usually three digits per group) for improved readability. The comma which is used as a punctuation mark after a number is, as in print, indistinguishable from a comma which occurs in the interior of a number.

The decimal point in a number is represented just like the period in a notational word. When the Dewey Decimal System is used for numbering sections and subsections of a text, a

number may contain several "decimal points" as, for example, "Section 2.3.1." As in the above example, a sentence-ending period is indistinguishable from a decimal point, as is also the case in print.

Remember that notational periods and commas must be used in such text.

3.3.6 Continental Usage

In many European countries, the decimal point is represented by a comma, and the comma is represented by a period. In NUBS, the same substitutions can be made, thereby preserving the "continental" character of the number.

3.3.7 Roman Numerals

Roman numerals are notational and require the notational indicator. The case (lower or upper) of a Roman numeral should be preserved in braille.

If a letter is attached to a Roman numeral either as a prefix or as a suffix, use dot 5 to separate the prefix or suffix from the Roman numeral to which it is attached.

3.4 Accent Marks, Diacritics, Foreign Languages

Unlike many foreign languages, the English language does not use accent marks. However, many foreign-language words and phrases have found their way into the English language and have become a part thereof; they are said to be anglicized words or anglicized phrases. Such anglicized words and phrases frequently contain accented letters. Different foreign languages feature different sets of accent marks, so that provision must be made for all of the accent marks that can occur in the words or phrases in foreign languages that are likely to infiltrate the English language. In the braille version of this document, accent marks are shown in Section 3.0.1 in the Basic Literary Symbol Set.

3.4.1 Accent Marks

NUBS is structured to address two different applications of accent marks.

The first of these applies to words and phrases that have come into regular use in the English language and are regarded as "anglicized," according to Rule 1, Section 6a(1), in Braille Formats. If such words or phrases contain accent marks, they are brailled using the rules presented in that section.

The second application of the use of accent marks involves "foreign" words or phrases–text that is not anglicized. The rules for words or phrases in this category are discussed in Section 3.4.3 where the use of "native accent marks" is addressed.

NUBS makes provision for the following accent marks to be used with "anglicized" words or phrases.

acute		hat	
bar		macron	
cedilla		ring	
circumflex		tilde	
diaeresis		umlaut	
grave			

The above set of accent marks is adequate for most purposes. In braille, they follow the letter which they affect, just as one would place an accent mark after having written the letter in print. The accented letter must be preceded by  if the accent mark is placed above the letter it affects, and by  if the accent is placed below the letter it affects. A word with accented letters is notational; it must be preceded by the notational indicator, and no contractions may be used within that word.

The literary code provides a generic accent mark which informs the reader that a letter is accented, but it provides no information as to the nature of the accent mark nor of its position above or below the letter that it affects. This information is readily available to the print reader.

NUBS makes this information available to braille readers who want or need it. However, the mechanism for providing this information is necessarily more cumbersome than is desirable by most people. Therefore, NUBS addresses this problem as follows:

A word which contains accented letters should be brailled according to the current literary code, using dot 4 as the accent mark indicator, at the site where it appears in print. However, the same word, using the NUBS mechanism for showing accented letters, should be presented on a Special Symbols Page at the beginning of the volume in which the word appears. If several words with accented letters appear throughout the volume, they should be collected and presented, alphabetically, using a two-column structure. The first column should show the form of the accented word as it appears in the body of the text. The second column would show the same word, using the NUBS mechanism for showing accented letters as described above. This format gives the braille reader access to all of the information about the accent marks that are used as the sighted reader has.

3.4.2 Diacritics

Diacritics is a system of writing whose purpose is to show the reader how a word is pronounced. It shows how a word is divided into syllables, which syllable has a primary stress and which one as a secondary stress when spoken, and how each symbol within a syllable is associated with the sound it represents.

There are various devices for showing how a diacritic word is enclosed, how the word is divided into syllables, different devices for showing stress, etc.

Diacritic words are separated from ordinary text by various forms of enclosure. These enclosures are: parentheses, brackets, slash and backslash, vertical bars and angle brackets.

Stress is shown by capitalization, by change of font, or by the use of actual stress marks.

Pronunciation is shown by the use of letters from the English alphabet, by modifiers, and by actually augmenting the English alphabet by a few additional letters.

A word that contains diacritics is notational; it must be preceded by the notational indicator and no contractions may be used in that word. Furthermore, words containing diacritics may not be divided between braille lines.

Since the method for showing syllable division in print is not standard, use the hyphen in NUBS to show syllable division regardless of the method used in print.

Various accent marks are used in print for indicating the pronunciation of vowels or vowel combinations. All these accent marks have representation in NUBS and they are all listed in Section 3.0.1, in the Basic Literary Symbol Set.

Print uses various methods for showing that two letters act as one. These are called *ligatures*. To show a ligature, a through-bar or an under-bar is used to tie two letters together. Often, a true ligature is used in which the two participating letters are printed so as to touch each other. A through-bar can be shown using the NUBS overstrike indicator, and an under-bar can be shown using the NUBS method for indicating a modifier beneath a letter. NUBS also offers a ligature indicator whose representation is:

ligature indicator

The ligature indicator has the effect of tying the next two letters together and is placed before the first of these two letters. A modifier or an accent mark may apply to either of these two letters or it may apply to the two-letter combination as a whole. If only one letter is modified, the mechanism for showing the modification is used for that letter only. If both letters are modified together, the indicator for showing modification above or below precedes the ligature indicator.

In diacritic notation, three letters augment the English alphabet. Their NUBS representations are:

breve  hooked n  schwa 

At the present time, there is no standard phonetic alphabet in braille. Therefore, it is recommended that you treat the phonetic alphabet like a foreign language; leave NUBS, use whatever phonetic alphabet is current, then return to NUBS.

3.4.3 Foreign Languages

NUBS makes provision for escaping into a foreign language in which the native accent marks of that language can be used.

If the excursion into the foreign language involves only one or two words, the first of the following indicators precedes each word. If the excursion into the foreign language involves

three or more words, the second of the following indicators is used before the first word of the foreign-language phrase:

begin foreign word ⠸⠸
begin foreign phrase ⠸⠸⠸

The following indicator is used to terminate a foreign-language phrase:

end foreign phrase ⠸⠄

If it is necessary to show a single foreign character, the indicator is:

for a single foreign character ⠸⠸

After the foreign-language word(s) or phrase, return to NUBS is automatic.

When in the foreign-language word or phrase, the accent marks native to that language are used–not the NUBS accent marks.
Note that the symbols ⠸⠸ and ⠸⠸⠸ do not reveal which foreign language has been entered, and therein lies a complication. In braille, in one language, a letter with its accent is represented by a single braille character. In another language, the same letter with the same accent is represented by a different braille character. For example, in French, e acute is represented by ⣿ while in Spanish, e acute is represented by ⣷ . Whereas a print reader is not told what language he has entered, a braille reader must have this information to interpret the native accented letters properly. Thus, we propose that a letter suggestive of the name of the language about to be entered be placed before the foreign-word or foreign-phrase indicator. The following letters are suggested for the most likely languages encountered:

French	f	Italian	i
German	g	Latin	l
Greek	k	Spanish	s
Hebrew	h		

3.5 Currency Signs

NUBS provides for the most common currency symbols. The prefix uniformly used before the relevant letters to represent symbols of currency is ⠈ . Here is the list of currency symbols which NUBS recognizes:

cent	⠈⠉	franc	⠈⠋
dollar	⠈⠎	pound sterling	⠈⠇
euro	⠈⠑	yen	⠈⠽

In the case of the dollar sign, *s* is used as the root because this is the letter which is slashed in print.

3.6 Fonts

In this section, *fonts* is used in a generic sense to include type variations such as capitalization, underlining, and colored printing, in addition to other standard fonts such as italic type, boldface type, and others.

Note that the method for beginning and ending a capitalized phrase follows the same pattern that is used for beginning and ending a phrase in any other font.

NUBS makes specific provision for the following fonts:

boldface			
begin phrase		one character	
end phrase		one word	
italics			
begin phrase		one character	
end phrase		one word	
underlined			
begin phrase		one character	
end phrase		one word	
upper case			
begin phrase		one letter	
end phrase		one word	

Since NUBS is extendable, other font indicators can be created using constructs analogous to those proposed above. Fonts that might be included are:

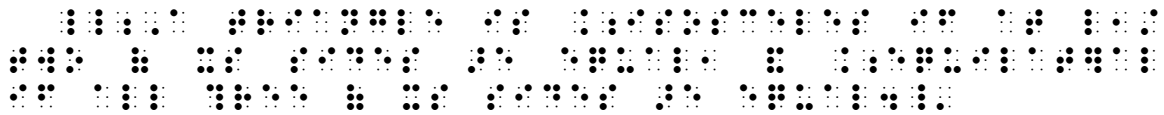
colored text
sanserif type
script font
transcriber-defined fonts as may be required

3.6.1 Typographic Conventions

In the full NUBS document, there is an excerpt from a computer manual (Turbo C) which should convince you of the need for these fonts. That excerpt is omitted in the condensed version.

Phrases in varying fonts may be nested to good effect. In some textbooks, definitions are printed in boldface type, with the term being defined printed in italic type without changing the boldface type. For example:

> **A triangle is *isosceles* if at least two of its sides are equal, and *equilateral* if all three of its sides are equal.**



The words "isosceles" and "equilateral" are italicized while remaining bold.

3.7 Further Extensions to the Literary Code

Here are some of the most common signs that you are likely to encounter from time to time in the general literature, presented in various categories.

3.7.1 Arrows

The most commonly occurring arrows are:

east		south	
north		west	
north-south		west-east	

The dot configurations of the one-way horizontal and vertical arrows suggest the direction in which the arrows point.

3.7.2 Comparison Signs

The NUBS symbols for the principal signs of comparison are as follows:

equals	
less than	
greater than	

The equals sign is the same as the contraction for "for." When confusion might occur because of this coincidence, the word "for" should be spelled out, as, for example, in

2 for $5 

rather than 

Similarly, the notational indicator must be used when the full cell represents the equals sign rather than the contraction for "for." For example:

5280 feet = 1 mile 

rather than 

Note the left-to-right symmetry of the "less than" and "greater than" constructs in braille as is also the case in print.

3.7.3 Grouping Signs

The following are the most common grouping signs:

angle brackets
left  right 
braces
left  right 

brackets
left  right 
double quotes
left oriented  right oriented  unoriented 
parentheses
left  right 
single quotes
left oriented  right oriented  unoriented 
transcriber's enclosure
left  right 

Note the left-to-right symmetry in each pair of the braille grouping signs in conformance with the left-to-right symmetry of the corresponding print signs.

The parentheses listed above require the punctuation indicator when they are in contact with a narrative word.

The brackets listed above replace do not require the punctuation indicator.

The braces listed above never require the punctuation indicator.

In print, the angle brackets listed above are primarily used for enclosing e-mail addresses and for enclosing other keystrokes. They constitute a further extension of the literary code, and they never require the punctuation indicator.

The double and single quotes, both oriented and unoriented, replace those of the literary code. They never require the punctuation indicator.

The transcriber enclosure signs are indicators since they correspond to nothing in print.

3.7.4 Legal Signs

NUBS provides special symbols for the following legal graphics:

copyright 
credit 
prescription (Rx)
registered 
trademark 

The "credit" sign is formed from the letters C and R touching each other so as to form a single graphic symbol that occupies one print position.

The literary code represents the "copyright," the "registered," and the "trademark" signs by enclosing the relevant letters within parentheses. But this is only an approximation to the exact symbols. NUBS provides symbols which, when back-translated, will produce the exact symbol required.
The R in the "prescription" sign stands for "recipe." Since "registered" and "prescription" have the same root, they cannot have the same prefix.

3.7.5 Miscellaneous Signs

Nubs provides for the following commonly occurring signs:

ampersand		degree sign	
at sign		infinity	
bullet		percent sign	
caret		per mil sign	
crosshatch		vertical bar	

The "at" sign and the ampersand may not be replaced by the words "at" and "and."

The "bullet" may not be replaced by a dash.

The crosshatch is sometimes referred to as the hash mark or the octothorpe. It is also called the "number" sign or the "pound" sign.

The "degree" sign is inherently at the superscript position and therefore never requires the superscript indicator to establish that position and does not require the base-level indicator to return to the base level.

The "infinity" sign looks like the digit 8 on its side in print and is sometimes referred to as the "lazy 8."

3.7.6 Operation Signs

The most common signs of operation in arithmetic and algebra are:

asterisk		slash	
divided by		times cross	
minus		times dot	
plus			

When the plus sign appears in print, it must be represented by the symbol listed above. For the most part, the minus sign cannot be distinguished from the hyphen except by context. However, this is also true for sighted people, so that the blind and the sighted are subject to the same pitfalls. Since the hyphen is a conditional delimiter whereas the minus sign is not, the braille user encounters a problem in this regard which a sighted user does not face.

The asterisk is used as the multiplication operator in computer programming and related text.

When multiplying two variables or two enclosed expressions, the multiplication operator is usually omitted altogether, as in *xy* or *(a+d)(b+c).*

In expressions which are contained in computer programs, the asterisk is most commonly used as the multiplication operator, and is never assumed to be implicitly present as in algebra.

In computer programs, the slash is most commonly used as the division operator; the dividend and the divisor are enclosed within parentheses as necessary.

Operators are more fully discussed in Chapter 5 and less common additional operators are also introduced there.

3.7.7 Reference Signs

NUBS offers the following reference signs:

asterisk		pointing finger	
dagger		reference indicator	
double dagger		section sign	
paragraph sign			

NUBS does not put reference signs in the superscript position as is almost always the case in print. Instead, the reference sign is always preceded by the reference indicator  to alert the braille user that there is a footnote or a marginal note on the braille page flagged by the same reference sign.

The mnemonic mechanism for forming the roots of the reference signs is as follows: Take the first letter of the sign's name and add dot 6. Thus, *p* is the first letter of "paragraph" and, when dot 6 is added, the root becomes  . According to this mnemonic, the root for "section" becomes  , for "dagger" it becomes  , and for "finger" it becomes  . Since the first letters of "dagger" and "double dagger" are identical, we use the letter *g* for "double dagger," and the root becomes  .

"Asterisk" is an exception. Other than being a reference sign, the asterisk serves many other functions, and we wish to use the same sign for the asterisk throughout NUBS.

Print uses many more reference signs than are provided by NUBS, and these are often referred to as "dingbats." Because of the extendability of NUBS, these dingbats can be created if required using the same prefix as is used for the reference signs above.

3.7.8 Simple Fractions

In print, the basic structure of a fraction consists of a ruled line called the ***fraction bar***, above which is the part of the fraction called the ***numerator***, and below which is the part of the fraction called the ***denominator***. The numerator and the denominator are frequently referred to as the ***terms*** of the fraction. Accordingly, the numerator is sometimes called the ***upper term***

and the denominator is sometimes called the *lower term* of the fraction. The length of the fraction bar is the same as the length of the longer term. If the length of the other term is shorter, it is generally centered on the fraction bar in print. Simple fractions are, by far, the most common.

In NUBS, fractions can be represented in two ways, linearly and spatially. Here we present the linear method.

When represented linearly, the characters which convey the information contained in a fraction occupy consecutive cells along the braille line. The symbol which represents the fraction bar is called the *linear fraction bar.*

When representing a linear fraction, NUBS uses the following symbols:

begin-fraction indicator ⠹
end-fraction indicator ⠼
linear fraction bar ⠌

A fraction is notational and, when embedded in surrounding text, must be introduced by the notational indicator ⠹ . Every fraction, without exception, must be enclosed within fraction indicators to mark the beginning and the end of the fraction. This requirement implements one of the basic design principles in NUBS, namely, the *just-in-time principle.* In the case of the fractions, the reader must know when a fraction has begun and when it has ended. A sighted person has this information just by looking at the fraction.

The current literary code provides only for fractions that contain numbers. However, as the following examples show, fractions may contain numbers, letters, operation signs, radicals, and any combination of these.

EXAMPLES (of simple fractions)

1 $\frac{1}{3}$ ⠹⠼⠁⠌⠉⠼

2 $\frac{2a}{3b}$ ⠹⠼⠃⠁⠌⠉⠃⠼

3 $\frac{5+4+6+5}{4}$ ⠹⠼⠑⠬⠙⠬⠋⠬⠑⠌⠙⠼

4 rate $= \frac{\text{distance}}{\text{time}}$ ⠗⠁⠞⠑ ⠶ ⠹⠙⠊⠎⠞⠁⠝⠉⠑⠌⠞⠊⠍⠑⠼

These examples are intended to demonstrate the variety of notation that can be contained in a simple fraction. We think of the first three fractions as being embedded in surrounding text and therefore require the notational indicator. We think of the last two examples as displayed text and thus do not require any notational indicator. Since Example 4 is displayed text, it is notational; none of the characters therein may be interpreted as contractions.

3.7.8.1 Quasi-Vertical Fractions

If a simple fraction is embedded within a print line in which the other characters are otherwise printed one after another in a straight line, the vertical structure of the simple fraction will result in considerable white space above and below the printed line. To minimize this white space, many typesetters will tilt the fraction bar so that it looks like a slash. They then print the numerator of the fraction near the top of this tilted fraction bar, and print the denominator of the fraction near the bottom of this tilted fraction bar. To reduce the amount of while space still further, they will, in addition, print the terms of the fraction in smaller type than is used for the other parts of the line. This reduces the length of the tilted fraction bar. Simple fractions that are printed using one or both of these methods are called *quasi-vertical fractions.*

NUBS does not distinguish between simple fractions with a horizontal fraction bar and a simple fraction that is quasi-vertical because there in no notational difference between the two forms, and white space is not an issue in braille. Fraction indicators must still be used, but the quasi-vertical fraction bar is represented by the linear fraction bar ⠌ . Thus, in a back-translation, the tilt of the fraction bar will be lost.

EXAMPLES (of quasi-vertical fractions)

1 $a+b/c+d$ ⠰⠁⠬⠃⠸⠌⠉⠬⠙⠰

2 $3\,x/y$ ⠼⠉⠼⠭⠸⠌⠽

In Example 1, the numerator is printed near the top of the tilted fraction bar, and the denominator is printed near its bottom. In Example 2, the fraction bar is also tilted, but this time the terms of the fraction are printed in smaller type.

3.7.8.3 The Slash

The symbol for the slash is:

slash ⠸⠌

Sometimes a typesetter may tilt the fraction bar without repositioning the terms of the fraction and without reducing the size of the type. Because the slash has so many other uses, NUBS does not recognize such text as a fraction. Consider the expression:

a+b/c+d ⠰⠁⠬⠃⠸⠌⠉⠬⠙⠰

It may be tempting to regard this text as a fraction whose numerator is *a+b* and whose denominator is *c+d.* But this interpretation would be mathematically incorrect. The correct mathematical interpretation is:

$a+\frac{b}{c}+d$ $\quad$ a plus (b over c) plus d

By treating constructs containing the slash in the manner just described, a sighted reader and a braille reader have the same opportunity of avoiding or of falling into the same trap.

RULES (regarding the slash)

a *Even when it appears "clear" that the slash is a substitute fraction bar, NUBS does not recognize such notation as a fraction unless other typesetting mechanisms attest to a quasi-vertical fraction.*

b *When a fraction is known to be quasi-vertical, its fraction bar, which looks like a slash in print, must nevertheless be rendered as a linear fraction bar in braille.*

c *In all other situations, the slash must be treated either as a sign of operation or as a punctuation mark.*

EXCEPTION See RULES (regarding mixed numbers: Section 3.7.8.4)

The slash has many uses other than as a substitute fraction bar. It is used between the components of a date. It is used between the components of model numbers, serial numbers, catalog numbers, and similar notation. It is used in conjunction with the asterisk to initiate and terminate a comment statement in the C language and in other modern programming languages. It is used as an informal substitute for the word "or" in many contexts such as in I/O, X-on/X-off, and/or, and he/she.

EXAMPLES (showing some uses of the slash)

1 1/3 ⠼⠁⠌⠉

2 I/(n) ⠠⠊⠌⠐⠣⠝⠐⠜

3 6/28/95 ⠼⠋⠌⠼⠃⠓⠌⠼⠊⠑

Example 1 shows a configuration in which the slash is a substitute fraction bar. However, no enclosing fraction indicators are used in this case, in accordance with NUBS rules.
The notation in Example 2 is not a fraction and is not represented as such. The transcriber is not expected to know what the notation means. The reader will probably rely on an explanation of this notation presented earlier in the text to assign a meaning to it.

In Example 3, the components of a date are separated by slashes.

3.7.8.4 Mixed Numbers

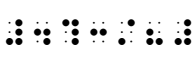
A *mixed number* consists of an integral part followed by a fractional part.

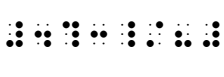
RULES (regarding mixed numbers)

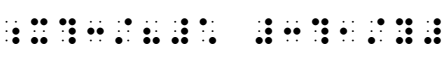
a *A mixed number consists of an integral part followed by a fractional part. In NUBS, the fractional part must be enclosed within fraction indicators.*

b *In the fractional part of a mixed number, a horizontal fraction bar in print must be represented by a linear fraction bar in braille, and a slash in print must be represented by a slash in braille.*

EXAMPLES (regarding mixed numbers)

1 $4\frac{3}{8}$ 

2 4 3/8 

3 $x\frac{3}{8}$, $3\frac{1}{y}$ 

The first two examples represent the same mixed number. In the first example, the fraction bar is horizontal; in the second, it is the slash.

The two expressions in Example 3 are not mixed numbers; both contain letters.

3.7.8.5 Graphic Symbols for One-Half and One-Fourth

One key on a standard typewriter contains two graphics: a graphic for *one-half* (without the shift key) and a graphic for *one-fourth* (with the shift key). These graphics occupy the space of one print character. Two-cell symbols have been devised for these graphics as follows:

the one-fourth graphic  the one-half graphic 

These symbols must be used only when displaying, describing, or discussing the layout of a typewriter keyboard.

3.7.9 Simple Radicals

NUBS offers the following symbols related to radicals:

radical sign: with vinculum ($\sqrt{\ }$)  without vinculum ($\surd$) 

termination indicator 

The radical sign, without further modification, is by far the most commonly encountered form. The radical sign is frequently referred to as the ***square root sign.*** Even the simplest calculators feature a square root key. Attached to the radical sign, and extending to the right to cover all the notation over which the radical sign is effective, is a horizontal bar called the ***vinculum.*** The notation under the vinculum is called the ***radicand.*** The notation that includes

the radical sign and the radicand is called the ***radical expression.*** A radical expression is notational.

The vinculum is not represented in braille. Instead, a ***termination indicator*** is supplied to show exactly where the vinculum ends. It is then implied that all the notation between the radical sign and the termination indicator is covered by the vinculum.

A radical sign without a vinculum is rare although it is sometimes used as the ***check mark.***

RULES (regarding the representation of a radical expression)

a ***The radical sign must be the first symbol of the radical expression.*** The radical sign is then followed by the radicand.

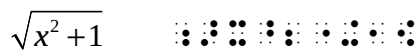
b ***The termination indicator must be used after the last character under the vinculum.***

c ***When there is no vinculum, NUBS provides an alternative radical sign.***

EXAMPLES (regarding simple radical expressions)

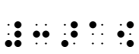
1 $\sqrt{2}$ 

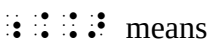
2 $\sqrt{x+y}$ 

3 $\sqrt{x^2+1}$ 

4 $\sqrt{x^2+y^2}$ 

5 $\sqrt{\frac{x}{y}}$ 

6 $3\sqrt{a}$ 

7 The symbol  means "square root."

8 $\sqrt{(x+y)}$ 

The expressions in Examples 1 through 6 are intended to show that there is no restriction on the notation that forms the radicand. In Examples 7 and 8, the radical sign has no vinculum.

3.7.10 Simple Subscripts and Superscripts

NUBS offers the following indicators related to simple subscripts and superscripts:

base level  subscript  superscript

Note how suggestive these indicators are for indicating their respective level changes.

The term ***subscript*** means "written lower" and the term ***superscript*** mean "written higher". These are accurate descriptions of the positions of such notation relative to the base line. Subscripts and superscripts show up frequently in the general literature. H_2O, the chemical formula for water, and CO_2, the chemical formula for carbon dioxide, are good examples.

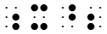
In print, subscripts and superscripts are written to the right of the notation to which they apply. Not only are subscripts and superscripts displaced from the base line, but they are generally printed in a reduced type size. This reduced type size is not an inherent part of the notation. It is done to reduce the amount of white space above and below the base line which contains them.

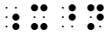
A subscript or a superscript may be as simple as a letter, a digit, or some other single symbol, or it may be a more complicated expression, including the possibility of fractions, radicals, or both. Braille is a system in which symbols are written linearly, one after another. It is not practical or feasible to write subscripts or superscripts at levels different from that of the base line, as is done in print. It is therefore necessary to devise symbols which indicate a level change rather than actually changing the level at which the notation is brailled. Such symbols are called *level indicators.* A level indicator does not represent a symbol that can be printed; rather, it offers important and in-time information about the disposition of the print symbols which it affects.

RULE (for level indicators)

Once a level has been established by one of the level indicators listed above, that level remains in effect until it is changed either by another level indicator or by a space.

EXAMPLES (illustrating the use of subscripts and superscripts)

1 x^2 

2 x^n 

3 $y^{1/2}$

4 $c^{\sqrt{2}}$

5 a_{pq}

6 $a_{pq} + b_{pq}$

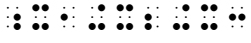
7 $x^2 + y^2 = 16$

8 35 ft^2

3.7.10.2 Numeric Subscripts

NUBS provides an abbreviated notation for numeric subscripts. This method dispenses with the subscript indicator, and the reader is returned to the base level when the number has ended. To qualify for this abbreviated notation, a number may contain not only digits, but commas and a decimal point as well. This notational abbreviation is available only when a number is a subscript to a letter; it does not apply to numbers which are subscript to other symbols. The letter to which a number is subscript may be in any case, an in any font, or from any alphabet. However, the letter to which the number is a subscript must be at the base level.

EXAMPLES (of numeric subscripts)

1 $x_1+x_2+x_3$ 

2 $a_{11}x_1+a_{12}x_2+ \ldots +a_{1n}x_n = b_1$

3 $a_{n1}x_1+a_{n2}x_2+ \ldots +a_{nn}x_n = b_n$

(The numbers which follow *x* are subscripts to *x*, but the numbers which follow *n* are at the same level as *n*.)

Numbers which follow a letter are not always subscript to that letter. This is particularly true in the case of model and serial numbers and in the identifiers in most programming languages. Only when the letter and the number are at the base level, the number, in NUBS, must be preceded by the numeric indicator, ⠼ . Note that this indicator differs from the one used in the 1972 Nemeth Code.

EXAMPLE (of numbers which follow letters but which are not subscripts)

4 Vitamin B12

(The number follows the letter at the base level.)

3.8 Contractions, Sequencing, Hyphenation

3.8.1 Contractions

The official literary code contains many anomalies regarding the use of contractions. NUBS does not attempt to resolve those anomalies. It regards the contraction system of the official literary code as a "black box" whose insides is not the domain of NUBS.

We make the following minor modifications:

a The contractions for "to," "into" and "by" may not be joined to a notational word.

b Do not contract "for" when it might be mistaken for the equals sign; use the notational indicator before the equals sign when it might be mistaken for "for".

3.8.2 Sequencing

Sequencing is the Grade-2 feature which permits certain words to be written without a space in braille even though there is a space between those words in print. The purpose of sequencing in braille is to save space.

Words that could be sequenced were:

"and," "for," "of," "the" and "with".

Furthermore, any of these words could be followed without a space by the word "a". In addition, the words

"to," "into" and "by"

could be written without a space next to whatever word followed.

However, at one time, Grade-2 braille contained a "natural pause" rule which disallowed sequencing when, if permitted, sequencing would destroy the sense of the sentence. Here is an example of such a situation:

He was passed by while others were noticed.

In this example, connecting "by" to "while" via sequencing would destroy the sense of the sentence, so sequencing was disallowed. However, the "natural pause" rule is difficult to implement on a computer, so that the rule has now been abolished.

Sequencing has a tendency to destroy the orthography of the English language which no code or system should do. However, NUBS is committed to preserving Grade 2. Therefore, NUBS will permit sequencing so as to leave Grade 2 intact. Nevertheless, here is a suggestion for partially reviving the "natural pause" rule which can easily be implemented by a table lookup. The words that can be sequenced become row headings in the table. The same words in the same order also become column headings in the table. We think of the words in the row headings as the "first word" and the words in the column headings as the "second word." If the first word can be properly sequenced with the second word, the table contains the letter X at the intersection of the row containing the first word and the column containing the second word. If it is inappropriate to sequence the first word with the second word, the intersection is blank. Here is the table:

	and	for	of	the	with	to	into	by	a
and		X	X	X	X				X
for				X					X
of				X					X
the									

with	X	X
to	X	X
into	X	X
by	X	X
a		

These rules, as previously asserted, would only partially restore the "natural pause" rule. The example above would not be helped by this table. But phrases like "to and fro" and "by and large" would.

The following sentence definitely would.

> He came to to see his mother standing over him.

⠠⠓⠑ ⠉⠁⠍⠑ ⠞⠕ ⠞⠕ ⠎⠑⠑ ⠓⠊⠎ ⠍⠕⠞⠓⠑⠗ ⠎⠞⠁⠝⠙⠊⠝⠛ ⠕⠧⠑⠗ ⠓⠊⠍⠲

3.8.3 Hyphenation

NUBS recommends that, for the most part, hyphenation be avoided. Experience has taught us that space is rarely saved by maximum hyphenation. It is indeed rare that a page can be saved in a braille volume as a result of hyphenating, and rarer still, in the case of two-sided embossing, that a sheet of paper will be saved by hyphenating in a volume of braille.

A hard hyphen is one that is present in the print text, as opposed to a soft hyphen which refers to a hyphen created by the transcriber when hyphenating a word. It is permissible to hyphenate at a hard hyphen if the braille line breaks at such a place.

3.8.3.1 Prefixes

There are, however, certain situations in which hyphenation is justified because, by not hyphenating, the ends of the braille lines would be too jagged. Below is a list of prefixes, common in the English language, collected over a period of time, after which hyphenation is justified because of the length of the prefix.

> aero ante anthro anti auto cardio circum contra counter electro extra
> hemo helio hetero hydro hyper hyper hypo infra intra mega micro
> multi neuro ortho photo physio poly proto pseudo psycho retro semi
> super tele thermo tran trans ultra

No doubt, there are other similar prefixes that are not included in the above list and, if recognized, may be added. Note that prefixes like "bio" and "inter" are not included. I believe that prefixes that require three cells or less do not qualify as candidates for hyphenation.

3.8.3.2 Suffixes

There are also a few suffixes that merit consideration as candidates for hyphenation. Some of these are:

> bilities bility cial cially cies hood ically

liest liness manship ties tious tive tory

As with prefixes, you will recognize other similar suffixes when you meet them. Only those which require four or more braille characters warrant consideration as candidates for hyphenation.

Do not hyphenate if the first character on the next line will be ⠨ or ⠰ . If dot 6 is the first character, the reader may take the letter that follows as upper case. If dots 56 form the first character, the reader may interpret the word that follows as notational.

3.8.3.3 Compound Words

Many words in the English language are a combination of words which were once separate or hyphenated but which have evolved into a single word. Typical words of this kind, called "compound words," are:

airplane, beekeeper, cupboard, daredevil, endpoint, farreaching, getaway, headhunter, icebreaker, jackhammer, keystroke, lipstick, masterpiece, nearsighted, overemphasize, payroll, quicksilver, racehorse, seascape, tapemeasure, underdeveloped, viewpoint, waterfront, yellowbird, zipperpull

Such words may be hyphenated between components of the word. Contractions, if available, may not be used to bridge across components, as is also the rule in the official literary code.

In all the above cases–prefixes, suffixes, and compound words–hyphenation is discretionary, not mandatory.

3.9 Other Alphabets, Special Letters

3.9.1 The Greek Alphabet

Not only are Greek letters used for notational purposes in mathematics and the physical sciences, they also occur in everyday literature. The names of fraternities and sororities are composed of Greek letters. The letter π (pi) is used when giving the formula for the circumference or the area of a circle. The letter σ (sigma) is used in the social sciences when giving the standard deviation of a set of data. The letter μ (mu) is used as the prefix meaning "micro" (a millionth) with units of measurement as in μg–microgram.

NUBS makes specific provision for the Greek alphabet, both lower-case and upper-case letters. NUBS uses the prefixes listed below to distinguish between a Greek lower-case letter and a Greek upper-case letter.

prefix for lower-case Greek letters ⠨
prefix for upper-case Greek letters ⠸

The roots for the Greek letters are those of the International Greek Alphabet devised under the auspices of UNESCO**:** I did not make the assignments. In the Basic Literary Symbol Set of this chapter, the representation for both upper-case and lower-case Greek letters is shown. In this condensed version of NUBS, lower-case and upper-case Greek letters are shown in Sections 3.0.7 and 3.0.8 on print page 15.

3.9.2 Other Alphabets

The use of alphabets other than the Greek alphabet for notational purposes is rare. Sometimes, a work is translated from a foreign language and the letters of that language that are used for notational purposes are retained in the translation. Thus, a work translated from the German may retain the Gothic (old German) letters used for notational purposes in the original. The same may be true of a work translated from the Russian which uses Cyrillic letters for notational purposes. The 1972 Nemeth Code made provision for the Gothic, Cyrillic and Hebrew alphabets. Although NUBS does not make specific provision for those alphabets, prefixes can easily be assigned for the letters of such alphabets as the need arises, because of the extendability of NUBS.

3.9.3 Special Letters

Letters from various alphabets are used for notational purposes; they are listed and discussed in Chapter 5.

INDEX FOR THE LITERARY SECTION.

In the full version of NUBS, an index is presented at this point because Chapters 0, 1, 2, and 3 constitute the Literary Section of NUBS. Only those aspects of mathematics that are common to general literature, such as those discussed in Section 3.7, are presented in this literary unit. The remaining chapters deal with the more scientific aspects of the proposed System.

Therefore, Chapters 0 through 3 are being treated as a self-contained entity, so that those who are interested only in the literary aspects of the System can limit their study to these chapters. Those who deal with more technical aspects of the System will want to investigate Chapters 4 and 5. At the end of Chapter 5, there is a global index that covers Chapters 0 through 5.

Each entry in the index includes the section number and the print page number. In the braille version, this is followed by an additional notation to provide the braille volume and

braille page number. For example a reference of: ANGLE BRACKETS 3.2.3 49 indicates that angle brackets are addressed in Section 3.2.3 on print page 49. Since the indexed item may not occur at the beginning of the referenced section, the print page listed may not include the start of that section.

This introduction to the Literary Index is included to acquaint the reader with the format used in the index. In the print document, section numbers and print page references are given with each entry. In the braille edition, these two references are followed (in parentheses) by an addition reference to the braille volume and braille page where that index entry can be found.

CHAPTER 4
COMPUTATIONAL SCHEMES

4.0 Chapter 4 Reference List

Description	Symbol
begin-cancellation indicator	
end-cancellation indicator	
first component of ruled line	
general component of ruled line	
one-character cancellation indicator	
single-cell representation of a two-digit number	

10 11 12 13 14 15 16 17 18

4.1 Recognizing a Computational Scheme

A computational scheme can be recognized by its two-dimensional disposition on the printed page and the vertical alignment of its elements into columns clearly intended for computation. A fraction, although its terms may be vertically aligned, is not a computational scheme because the alignment is not for the purpose of computation. Thus, a computational scheme is displayed text which must be treated as spatial material. All the examples in this chapter are computational schemes.

4.2 Format Requirements

The basic NUBS document devotes about two print pages to the issue of format for computational schemes. This is omitted in the condensed version of NUBS. Consult the full document if you are interested in these format issues.

4.3 Help for Young Children

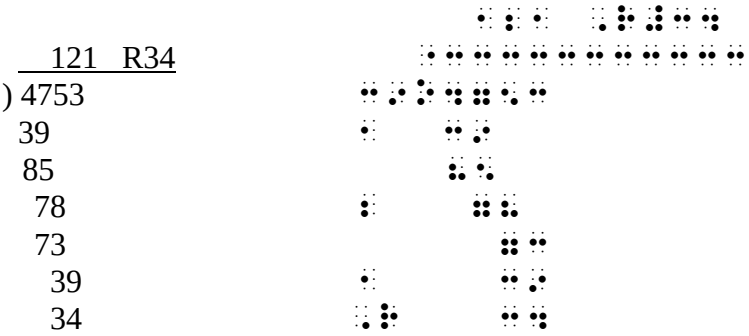
4.3.1 Column Separation

This section offer a suggestion for teachers of the blind to prevent a student from wandering up or down in moving across a row in a computational scheme. An example is given if the full NUBS document.

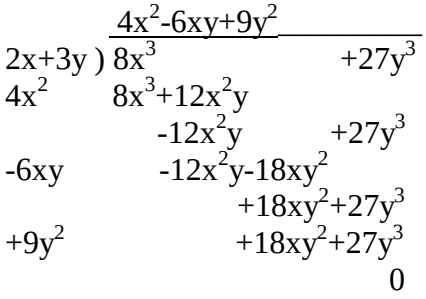
4.3.2 Avoiding Paper Creep

People who use a brailler in the computation of a long division example (strongly recommended) frequently experience the "creeping paper" problem. As the paper is rolled up to the quotient line to enter the latest digit or the latest algebraic term and then rolled back down to the current work area, there is a tendency for the paper to "creep" so that the new braille entry is too close to the previous braille entry for comfortable reading. With a repetition of this "roll up and roll down" activity as each new digit or term is developed, which is also time-consuming, this "creeping" behavior finally causes the new braille and the old braille to merge so that both are illegible. To solve this problem, it is suggested that each new digit or term, as it is developed, be entered against the left margin abreast of the partial product about to be generated. Thus, the paper is always advanced and never rolled back. When the computation is complete, the digits or terms of the quotient can be read from top to bottom of the paper against its left margin. The final remainder is also available on the last line of the computation. If desired, these can now be transferred to the quotient line. The student should be informed that this is a "braille-only" ploy to avoid "paper creep." If the work is to be handed to a sighted teacher who knows braille, or if it is to be turned into writing by a vision teacher for submission to a subject-matter teacher, the quotient line must be supplied. Notice the NUBS method for showing the format of a long-division problem.

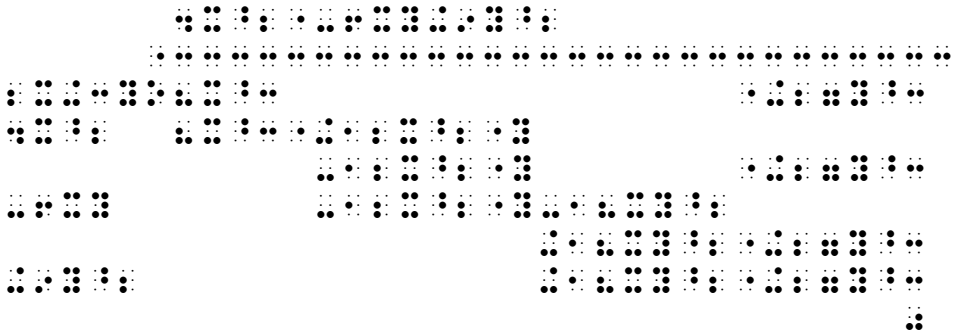
EXAMPLES (avoiding "paper creep")



```
    121  R34
39 ) 4753
1    39
     85
2     78
      73
1      39
R      34
```



$$\begin{array}{l} \qquad\qquad \underline{4x^2-6xy+9y^2\qquad\qquad\qquad} \\ 2x+3y\ )\ 8x^3 \qquad\qquad\qquad +27y^3 \\ 4x^2 \qquad 8x^3+12x^2y \\ \qquad\qquad\ -12x^2y \qquad\quad +27y^3 \\ -6xy \qquad\ -12x^2y-18xy^2 \\ \qquad\qquad\quad +18xy^2+27y^3 \\ +9y^2 \qquad\qquad +18xy^2+27y^3 \\ \qquad\qquad\qquad\qquad\quad 0 \end{array}$$



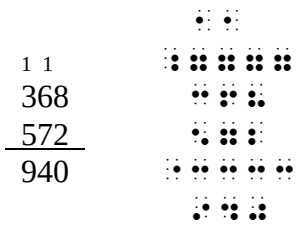
These two examples are fictitious homework papers of fictitious blind children. One is a third or fourth grader studying arithmetic; the other is an eighth or ninth grader studying elementary algebra. They both place their work against the left margin since there is no reason to indent. They omit the ruled lines that are usually present in a worked-out textbook presentation except for the ruled line above the dividend. As each new digit is determined in the arithmetic example or as each new term is determined in the algebra example, it is placed against the left margin instead of on the quotient line as sighted children are taught to do. They have been told that this is a "braille trick" to avoid rolling the paper up and down. Upon completing the computation, the digits or terms are read from top to bottom and copied into the quotient line.

4.4 Carried-Number Line in Addition

In print, carried numbers are usually shown as reduced in size and appear directly above the top entry of the column to which they apply. In this System, we devise a "carried-number line." Each component of the "carried-number line," except the first, consists of dots 2356.

The first component of the carried-number line consists of dots 56. Its length is the same as the length of the ruled line at the bottom of the addition scheme.

EXAMPLE (showing a carried-number line)



Since the carried-number line is longer at both ends than the terms that participate in the addition example, and since its first component is different from the remaining components, the carried-number line cannot be confused with a string of 7's as a part of the example.

4.5 Cancellation When Subtracting

This section is included in this condensed version because it offers unique insight into a real problem that math students now encounter. This new scheme has been tried on students in Orange County, Florida with amazing success!

When imparting to young children the skill of subtracting one number from another, an intermediate phase in the development of that skill is to have the children actually write down the numbers that play a role in a procedure called "renaming" (sometimes called "borrowing"). The form of this renaming procedure involves the cancellation of some numbers and their replacement by others. When the subtraction skill is well established, the renaming process is carried out mentally. Until then, the children must pass through this intermediate stage.

In print, we can, with one stroke (no pun intended), simultaneously show the extent of the cancellation and preserve the readability of what has been cancelled. In braille, such a feat is not easily achieved.

EXAMPLES (showing cancellation problems using the 1972 Nemeth Code)

1
```
 1456
-  123
```

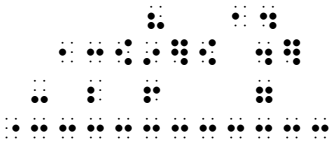
It is clear at a glance that Example 1 does not require any renaming since each of the digits being subtracted is less that the digit above it. The subtraction can be carried out easily.

2a
```
 1394
- 267
```

2b
```
  8 14
 1394
- 267
```

Example 2a does require some renaming. It can be seen at a glance that 7 cannot be subtracted from 4, so we need to "borrow" a 1 from the 9, changing it to a 8 and replacing the 4 by 14. In print, this is no problem in that the number 14 can be written above the 4 in smaller font size so that the alignment of the columns is maintained. This is shown, in print only, as Example 2b.

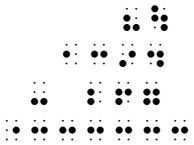
Example 2c below shows how, using the 1972 Nemeth Code, extra spaces and cancellation indicators are required in order to preserve the alignment.

2c 

This process is awkward and requires extra planning on the part of transcribers to assure that the columns line up properly. When several columns require renaming, this process grows to unacceptable complexity.

NUBS solves this cumbersome subtraction process by providing a clever technique. Fortunately, a two-digit replacement number in a subtraction scheme is always in the range of 10 to 18 inclusive. Therefore, we are able to contrive a "trick" for entering this two-digit number into one cell. We enter the unit's digit of this number as a dropped number, as usual. We now add dot 1 into the same cell to indicate that this is a two-digit number whose ten's place is 1. Thus the numbers from 10 to 18 would be represented in one cell as follows:

Using this scheme, the problem above would now be brailled as:

2b 

In NUBS, we do not do anything to cancel a digit. Simply brailling its replacement number on the braille line above it implies the cancellation of the digit. Thus, its form is not changed and it remains readable.

In the process of subtraction, replacement digits may be required at more that one level above the subtraction problem. This occurs when a "renamed digit" requires renaming as the process moves leftward across the subtraction columns.

When the renaming procedure has been completed, the actual subtraction is performed. In each column, the digit in the bottom number is subtracted from the topmost number in its

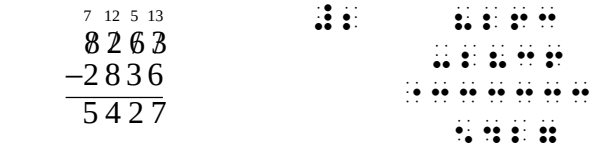
column. Because of the renaming that was done before the actual subtraction was undertaken, the subtraction will always be possible. In each column, the topmost entry will be either a compressed two-digit number or a single digit.

Some additional examples of this procedure are given in the next section.

Here is one example of this renaming scheme, taken from Section 4.5.1.

EXAMPLES (showing a revised cancellation technique for subtraction)

2



In this example, only one line of numbers above the minuend is required. If we number the columns 1 to 4 from right to left, we find that the compressed two-digit number 13 is in column 1 and the compressed two-digit number 12 is in column 3. We now can complete the problem by subtracting 6 from 13, 3 from 5, 8 from 12, and 2 from 7, entering each result in the remainder line. Since, after the renaming procedure, subtraction is possible in each column, we can even proceed to perform the individual subtractions from left to right, thereby avoiding backspacing when using the braillewriter.

4.6 Cancellation in General

Subtraction schemes do not have a monopoly on the use of cancellation. In arithmetic, cancellation is used in the process of reducing fractions to lowest terms, and in the multiplication or division of fractions. In algebra, cancellation is used for the same reasons; however, there we would be dealing with number-and-letter combinations rather than just with numbers as in arithmetic.

NUBS offers a mechanism for showing cancellation in which the extent of the cancellation is precise and the cancelled material remains readable. We provide the following cancellation indicators:

begin cancellation ⣿ end cancellation ⠾ one-character cancellation ⠿

RULES (for cancellation)

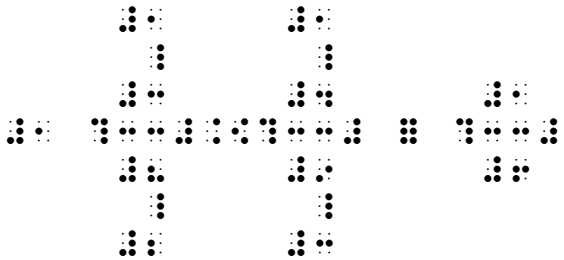
a *Text in which cancellation occurs must be presented in displayed form. Text containing cancellation is spatial; it must be separated from surrounding text by a blank line above and below it.*

b *If the text to be cancelled occupies one character, the one-character cancellation indicator must be placed above or below it, as appropriate. If the text to be cancelled occupies more than one cell, the begin-cancellation indicator and the end-cancellation indicator*

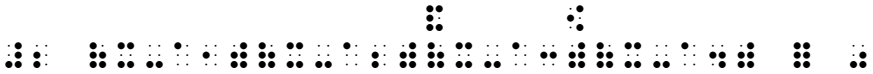
must be placed at the first and the last cell, respectively, of the text to be cancelled; these indicators should be placed above or below the text to be cancelled, as appropriate.

EXAMPLES (showing cancellation)

1 $\frac{\cancel{3}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}_{3}} = \frac{1}{6}$



2 $(x-a_1)(x-a_2)\cancel{(x-a_3)}(x-a_4) = 0$



Example 1 shows how two fractions are multiplied. The 3 in the first numerator cancels the 9 in the second denominator. These cancelled numbers are replaced by 1 and 3 respectively. The number 8 in the first denominator and the 4 in the second numerator have a common factor of 4. These numbers are cancelled and replaced by the result obtained by dividing each one by 4. The result is obtained by multiplying the two new numerators and the two new denominators.

Example 2 shows the product of four algebraic expressions in which the third factor has been cancelled without a replacement.

4.7 The Abacus and the Calculator

The abacus and the talking calculator have become staples in the math curriculum of blind children. When they are properly used, they yield results with the speed and accuracy that matches or even surpasses the speed and accuracy attained by sighted people. Therefore, they are valuable aids when put into the hands of blind users.

When the pencil-and-paper skills are acquired in the performance of arithmetic operations, they are easily transferred to analogous algebraic operations. The abacus and the calculator

are restricted to arithmetic operations only; they are useless in the performance of algebraic operations.

Therefore, access to the abacus and the calculator should be deferred until after the blind child has acquired the skills described above. He will then have both the skill and the knowledge of the algorithms underlying the basic arithmetic operations and the means for achieving the speed and accuracy required in everyday activities.

CHAPTER 5
EXTENSION TO MATHEMATICS

Mathematics is part of the natural sciences. These encompass the fields of mathematics, physics, chemistry, and the various branches in each of these fields. They exclude the life sciences and the humanities. There are no rigid boundaries among these fields, so that the notation in any of these fields will find its way into some other field. For example, the notational constructs in organic chemistry are frequently found in the literature that deals with microbiology. The symbols of mathematics are prominent in the field of physical chemistry.

This chapter concentrates on the extension to mathematics. We turn our attention to the field of mathematics first because its notation, to a large extent, permeates all of the other natural sciences.

In Chapter 3, we proposed a whole set of symbols that are not only mathematical, but which have found their way into the general literature. We also undertook a cursory analysis of fractions and radicals, and we addressed the basics of subscripts, superscripts and related matters.

We now include some symbols that were deliberately left out in earlier chapters. In those chapters we addressed the kind of text likely to be found in everyday reading material. Therefore, we presented only the basics in each covered category. Here we supply more details for each of those categories. We then go on to categories of a more technical nature which were not even mentioned in earlier chapters.

5. 0 Mathematics Symbol Set

The Mathematics Symbol Set contains symbols found in texts that are more technical. Symbols that are likely to be found in everyday texts are listed in the Basic Literary Symbol Set. The mathematics symbols listed below have been sorted into categories and the categories have been alphabetized.

Description	Braille
5.0.1 ARROWS (continued)	
northeast	
northwest	
northwest-southeast	
southeast	
southwest	
southwest-northeast	

5.0.2 ASCII CHARACTER SET (excludes letters and digits)

ampersand		left parenthesis	
apostrophe		percent sign	
asterisk		period	
at sign		plus	
backslash		question mark	
caret		quote	
colon		right angle bracket	
comma		right brace	
crosshatch		right bracket	
dollar sign		right parenthesis	
equals		semicolon	
exclamation		slash	
grave accent		space	(no dots)
hyphen		tilde	
left angle bracket		underscore	
left brace		vertical bar	
left bracket			

5.0.2.1 AUXILIARY NOTATION

begin upper-case passage		upper case, one letter	
end upper-case passage		upper case, one word	

5.0.3 CALCULUS AND ANALYSIS

absolute value		lower limit	
del		nabla	
double vertical bar		natural logarithm	
infinity		norm	
integral		partial derivative	
large boldface pi		prime	
large boldface sigma		upper limit	
limit		vertical bar	
logarithm			

5.0.4 COMPARISON SIGNS (continued)

approximately equal		is an element	
contains as an element		is a subset	
contains as a subset		is not an element	
does not contain the element		is not a subset	
does not include		less than or equal	
greater than or equal		not equal to	
implies		not greater than	
includes		not less than	

5.0.5 COMPUTER AND CALCULATOR

begin keytop legend	
begin screen display	
runover indicator	

5.0.6 FRACTIONS AND RADICALS (continued)

complexity indicator for fractions and radicals	
component of a radical ascender	
component of spatial fraction bar	
component of a spatial vinculum	
last component of radical ascender	
radical sign	
radical sign without vinculum	

5.0.7 FUNCTIONS AND THEIR ABBREVIATIONS

amplitude	amp	haversine	hav
antilogarithm	antilog	imaginary	im
arc	arc	infimum	inf

argument	arg	limit	lim
cologarithm	colog	logarithm	log
cosine + *i*sin	cis	lower limit	<u>lim</u>
coversine	covers	maximum	max
curl	curl	minimum	min
determinant	det	modulo	mod
dimension	dim	modulus	mod
divergence	div	natural logarithm	ln
error function	erf	rank	rnk
exponential	exp	real	re
exsecant	exsec	supremum	sup
gradient	grad	upper limit	$\overline{\text{lim}}$
		versine	vers

5.0.8 GEOMETRY

5.0.8.1 GEOMETRIC MODIFIERS

full line

half lines

- closed left
- closed right
- open left
- open right

line segments

- left closed, right closed
- left closed, right open
- left open, right closed
- left open, right open

5.0.8.2 GEOMETRIC SHAPES

angle		parallelogram	
arc, concave		pentagon	
arc, convex		quadrilteral	
circle		rectangle	
diamond		rhombus	
ellipse		right angle	
equilateral triangle		right triangle	
hexagon		square	
is parallel		star	
is perpendicular		trapezoid	
octagon		triangle	
oval			

5.0.9 GROUPING SIGNS (continued)

barred signs

braces

- left
- right

brackets

left right
boldface signs
brackets
left right
vertical bar
left and right
double vertical bars
left and right
extended signs
braces
left right
brackets
left right
parentheses
left right
transcriber enclosure
left right
vertical bar
left and right
half-brackets
lower left upper left
lower right upper right
vertical bar
left and right

5.0.10 INDICATORS (continued)

begin keytop legend
begin screen display
complexity indicator for
fractions and radicals
no boundary
overscript
overstrike
runover
shaded shape
solid shape
underscript

5.0.11 LOGIC AND SET THEORY

Cartesian product
contains the element
contains the subset
does not contain the element
does not contain the subset
empty set
for all
global Cartesian product
global intersection
global union
if and only if
intersection
is implied by
is not an element of
is not a subset of
is a proper subset of
is subset of
join
meet
there exists
there exists uniquely
union

implies

5.0.12 MODIFIERS

acute accent		left arrow	
asterisk		macron	
breve		right arrow	
circumflex		ring	
diaeresis		tilde	
dot		two-way arrow	
grave		umlaut	
hat			

5.0.13 OPERATION SIGNS (continued)

Cartesian product		meet	
global Cartesian product		minus or plus	
global intersection		plus or minus	
global union		union	
join			

5.0.14 PUNCTUATION (continued)

ellipsis, vertical

5.0.15. SPECIAL LETTERS

5.0.15.1 CROSSED

b		h	
d		λ (lambda)	

5.0.15.2 GLOBAL OPERATORS

global product		global sum	

5.0.15.3 HEBREW

aleph		beth	

5.0.15.4 MISCELLANEOUS

partial derivative

Weierstrass *p*

5.0.15.5 NUMBER SETS

complex		rational	
integers		real	
natural			

5.0.15.6 OBSOLETE GREEK

koph or koppa		sampi	

5.0.15.7 VARIANT GREEK

alpha	⠨⠁	sigma	⠨⠎
beta	⠨⠃	phi	⠨⠋
theta	⠨⠹		

5.0.16 TRIGONOMETRY

arccosecant	arccsc	inverse cosecant	csc^{-1}
arccosine	arcos	inverse cosine	cos^{-1}
arccotangent	arccot	inverse cotangent	cot^{-1}
arccotangent	arcctn	inverse cotangent	ctn^{-1}
arcsecant	arcsec	inverse hyperbolic cosecant	$csch^{-1}$
arcsine	arcsin	inverse hyperbolic cosine	$cosh^{-1}$
arctangent	arctan	inverse hyperbolic cotangent	$coth^{-1}$
cosecant	csc	inverse hyperbolic cotangent	$ctnh^{-1}$
cosine	cos	inverse hyperbolic secant	$sech^{-1}$
cotangent	cot	inverse hyperbolic sine	$sinh^{-1}$
cotangent	ctn	inverse hyperbolic tangent	$tanh^{-1}$
hyperbolic cosecant	csch	inverse secant	sec^{-1}
hyperbolic cosine	cosh	inverse sine	sin^{-1}
hyperbolic cotangent	coth	inverse tangent	tan^{-1}
hyperbolic cotangent	ctnh	radian	rad
hyperbolic secant	sech	secant	sec
hyperbolic sine	sinh	sine	sin
hyperbolic tangent	tanh	tangent	tan

5.1 Arrows (continued)

In Chapter 3 we showed and discussed only horizontal and vertical arrows. There are, however, oblique arrows which are found mostly in chemistry, and these have been listed in Section 5.0.1 of the Mathematics Symbol Set.

Arrows are also used as modifiers in vector notation. NUBS permits these arrows to be omitted if there is other evidence that the notation represents a vector. However, omission of such arrows is optional--not mandatory.

5.3 Calculus and Analysis

The signs that are closely related to the field of calculus and analysis are shown in Section 5.0.3 of the Mathematics Symbol Set.

The signs for vertical bar and double vertical bar belong to the category of Grouping Signs as well as to the category of Calculus and Analysis.

The root of the infinity sign is the digit 8, suggestive of the fact that it is an 8 on its side in print. It is sometimes referred to as the "lazy 8."

The integral sign frequently carries limits of integration as a subscript and a superscript. These sometimes have the appearance of underscript and overscript modifiers. However, the integral sign is too thin to support underscripts and overscripts. Therefore, despite appearances, these should be transcribed as subscripts and superscripts.

Contour integrals have various closed shapes superposed on them. When this occurs, the integral sign should be followed by the overstrike indicator ⠒⠿ which, in turn, should be followed by the superposed shape.

The letters in the three "limit" entries are printed in the host type as is the case for all function abbreviations. An alternative for the upper limit is the letters lim with an overscript bar. Similarly, the lower limit is represented by the letters lim with an underscript bar. All three limits may have other underscripts.

log and ln are function abbreviations printed in the host type. Most of the time, log without a subscript is understood to have 10 as its base. At other times, 10, *e*, or some other base may be attached as a subscript to log. ln without a subscript is always understood to have *e* as its base. A common transcriber error is to mistake ln for 1*n*. Note that both letters in ln are in the host type. If 1*n* were intended, *n*, being a variable, would be printed in the mathematical italic type. Noting this rather subtle distinction would avoid this error.

The principal use of the prime sign in calculus is to denote the order of the derivative. Thus, f ' (*x*) denotes the first derivative of f (*x*), f " (*x*) denotes the second derivative of f (*x*), etc. After the third derivative, it is customary to use Roman numbers enclosed within parentheses as superscripts to denote the order of the derivative. Thus $f^{(iv)}(x)$ denotes the fourth derivative of f (*x*).
All the function abbreviations in trigonometry are also prominent in calculus.

The partial derivative sign is sometimes referred to as "round d."

5.4 Comparison Signs (continued)

The basic comparison signs were presented in Chapter 3, Section 3.0.3. Other comparison signs, together with their tactile graphics, are presented in Section 5.0.4 of the Mathematics Symbol Set.

5.5 Computer and Calculator

Most modern arithmetic and algebra textbooks and computer and calculator manuals present a sequence of keystrokes by displaying a row of keytop legends. Legends other than digits are printed on a picture of a keytop. Digits are often displayed without being superposed on the picture of a keytop.

NUBS provides a method for showing a sequence of keystrokes when such a sequence is present in print. A keystroke which is the picture of a keytop with its legend is introduced in NUBS by the braille character ⠒⠶ . In other notational contexts, this character is either the termination indicator or the end-cancellation indicator. When it is the first character of a keystroke, it cannot be interpreted as a termination indicator. Neither can it be interpreted as

an end-cancellation indicator which is always placed either above or below the notation to be cancelled. Therefore, using this symbol to introduce a keystroke is unambiguous.

After this begin-keytop-legend indicator, we write the keytop legend using standard NUBS notation. A space follows the keytop legend before brailling the next keystroke. Keystrokes are notational and spatial and must be treated accordingly.

EXAMPLES (showing keystrokes)

1 Press [+/-] 

to change the sign of the number. (The legend on the keytop is +/- .)

2 2.41 [EE] 0 [÷] 7.4 [EE] 21 [=]

(A sequence of keystrokes in which some are numbers and some are not.)

The result of a calculation is displayed on the screen of the calculator. The symbol  is used in NUBS to introduce this display. In other contexts, this symbol is the begin-cancellation indicator. However, as such, it must be placed above or below the notation to be cancelled. Therefore, the proposed symbol cannot be confused with the begin-cancellation indicator. The "rectangular" shape of this symbol suggests the rectangular shape of the screen. The information displayed is presented in standard NUBS notation. Like a sequence of keystrokes, the screen display of a calculator is both notational and spatial.

EXAMPLE (showing the screen display of a calculator)

3 [3.2568 e-22] 

5.6 Delimiters (continued)

In Chapter 2, Section 2.3, we discussed delimiters and presented some examples. It is recommended that you review that section now. Here is further discussion and more examples.

The space is not a delimiter when it occurs between a function name or a function abbreviation and its argument. This holds even if the function name or the function abbreviation has subscripts, superscripts, overscripts, underscripts, modifiers, or any combination of these.

The space is not a delimiter when it occurs between a geometric shape and the enumeration of its points or vertices.

EXAMPLES (in which the space is not a delimiter)

1 $\cos x$

2 $\log_{10}$ 2.7183 . . .
(The space after $\log_{10}$ is not a delimiter even though log has a subscript.)

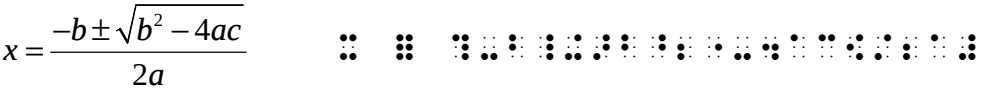
3 $\sin^{-1} \frac{1}{2} = \frac{p}{6}$ (None of the three spaces is a delimiter. Why?)

5.7 Fractions and Radicals (continued)

The NUBS basics for representing fractions and radicals were presented in Chapter 3; it is recommended that you review those topics at this time before proceeding to more advanced issues.

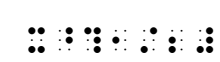
Other codes have attempted to avoid fraction indicators by enclosing the terms of the fraction within parentheses or other enclosures. However, if the numerator of a fraction is long and the denominator is short, you will not know that you are dealing with a fraction until you encounter the fraction bar near the end of the expression. The quadratic equation is a classic case in point:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



If there were a left parenthesis instead of a begin-fraction indicator in this expression, you would not know that you are dealing with a fraction until you encountered the fraction bar within two symbols from the end of the fraction. Your mental orientation to the notation at hand may be quite different when you know that you are dealing with a fraction than when you are coping with the same expression but without that information. Besides, using enclosures of any kind would be a violation of one of the NUBS principles and guidelines which mandates that no enclosures be used in braille, either real or phantom, when there are no enclosures in print.

$x^{\frac{1}{2}}$



shows that fractions can be in the superscript position. They can also be in the subscript position; they can be part or all of a radicand. On the other hand, fractions can contain subscripts, superscripts and radicals.

The slash, as a fraction bar or for other uses, is discussed in Chapter 3.

5.7.1 Fractions of Higher Order

It may happen that the numerator or the denominator of a fraction, or both, are themselves fractions or contain fractions. When this occurs, we are dealing with *higher-order fractions.*

A *simple fraction* is one in which neither the numerator nor the denominator is (or contains) a fraction. A simple fraction is said to be of order 0. For purposes of NUBS only, the order of a fraction is not increased if the only fractions that it contains are at the subscript level, the superscript level, or both.

A *complex fraction* is one in which the numerator, the denominator, or both are (or contain) only simple fractions. A complex fraction is said to be of order 1. The term "complex" is also used in a generic sense to apply to any fraction of order higher than 0. That is, "complex" is used as the opposite of "simple" when referring to fractions.

A *hypercomplex fraction* is one in which the numerator, the denominator, or both are (or contain) a complex fraction. A hypercomplex fraction is said to be of order 2.

In theory, it is possible to have fractions of any order whatever. As a practical matter, we frequently encounter complex fractions, we sometimes encounter hypercomplex fractions, and we almost never encounter any fractions of higher order than 2.

Since a fraction of higher order contains more than one fraction bar, it is necessary to identify the main fraction bar. In print, this is usually the fraction bar that is the longest in the entire construct, the one that is abreast of a reference label, a comparison sign, or some other sign of operation, or the one which is printed in type heavier than the others.

NUBS provides a *complexity indicator* to inform the reader of the complexity of the fraction with which he is dealing. This is another instance in which NUBS implements the *just-in-time principle.*

RULES (regarding the complexity of a fraction)

a *These rules apply only when representing a fraction linearly.*

b *The complexity indicator must not be used with simple fractions.*

c *The complexity indicator must be used once before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a complex (first order) fraction.*

d *The complexity indicator must be used twice before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a hypercomplex (second order) fraction.*

e *In general, in the case of a fraction of order n, the complexity indicator must be used n times before the begin-fraction indicator, the end-fraction indicator, and the fraction bar of such a fraction.* Fractions of order greater than 2 are almost never encountered.

EXAMPLES (regarding complex fractions)

$$\frac{\frac{3}{8}}{5}$$

1 

2 $1/2 \over 2\frac{2}{3}$ 

Example 1 is a complex fraction whose numerator is a simple fraction.

Example 2 contains a mixed number in its denominator. The fractional part of this mixed number is a simple fraction, causing the entire construct to be treated as a complex fraction.

5.7.2 Spatial Representation of Fractions

NUBS provides for the representation of fractions in a form which simulates their appearance in print. Fractions presented in this way are called ***spatial fractions,*** and the fraction bar in such fractions is called a ***spatial fraction bar.***

There are several situations in which it is appropriate to represent fractions spatially:

- When fractions are first introduced to young children and for a short while thereafter;
- When the structure of a complex fraction is more clearly displayed spatially than linearly;
- In the representation of a continued fraction;
- When performing cancellation or other computation with fractions.

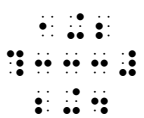
Although continued fractions are spatial, they are excluded from this discussion.

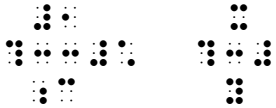
RULES (regarding spatial fractions) (omitted in this condensed version)

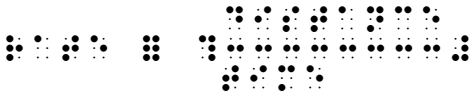
The rule regarding the centering of the terms in a spatial fraction and their possible runovers has been changed from the 1972 Nemeth Code. There are also some other minor changes from the 1972 Nemeth Code.

EXCEPTION
Fraction indicators must not be used in a continued fraction.

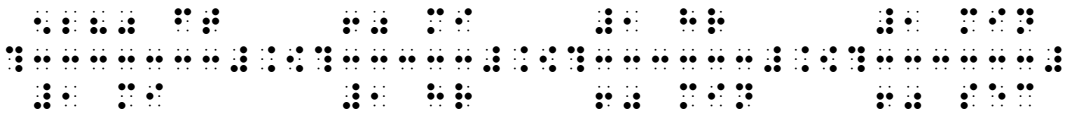
EXAMPLES (regarding spatial simple fractions)

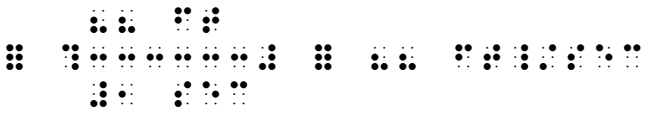
1 $\frac{1+2}{2+4}$ 

2 $\frac{1}{c}, \quad \frac{x}{y}$ 

3 $rate = \frac{distance}{time}$ 

4 $\frac{5280\ ft}{1\ mi} \text{ x } \frac{60\ mi}{1\ hr} \text{ x } \frac{1\ hr}{60\ min} \text{ x } \frac{1\ min}{60\ sec} = \frac{88\ ft}{1\ sec} = 88\ ft/sec$





Example 1 is the kind that might be encountered in an elementary grade when fractions are first being introduced to young children.

Example 2 shows how isolated digits and letters are handled.

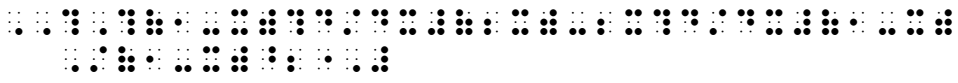
Example 3 consists of words. But since it is displayed, it is notational so that it contains no contractions.

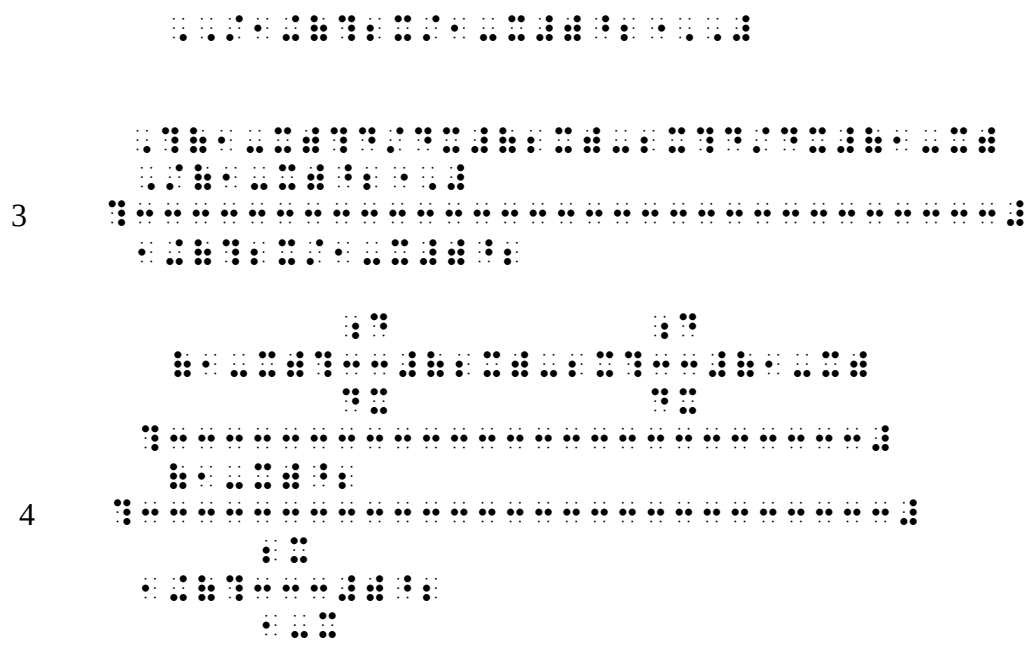
Example 4 is a typical example of a technique called "dimensional analysis." Putting all of this together, we find that 60 miles per hour (the second fraction) is the same as 88 feet per second.

EXAMPLES (regarding spatial complex fractions)

1 $\cfrac{1\frac{1}{4}}{1\frac{3}{5}}$ 

2 $$\frac{\frac{(1-x)\frac{d}{dx}(2x)-2x\frac{d}{dx}(1-x)}{(1-x)^2}}{1+\left(\frac{2x}{1-x}\right)^2}$$





3

4

Example 1 is the spatial representation of a hypercomplex fraction. The numerator is the quotient of two mixed numbers and so, in itself, is a complex fraction. This complex fraction is divided by 5, making the entire construct a hypercomplex fraction.

Examples 2, 3, and 4 are alternative ways of representing the same print information. In Example 2 the representation is entirely linear even though it is included in a group identified as spatial complex fractions. Example 3 is partly linear and partly spatial. Example 4 is entirely spatial.

When requested to find

$$\frac{d}{dx}\tan^{-1}\frac{2x}{1-x}$$

this notation is the first step in working out the solution to finding this derivative.

5.7.4 Indexed Radicals

A radical is said to be ***indexed*** if it carries a superscript. If the superscript is 3, we are dealing with the cube root; if the superscript is *n*, we are dealing with the n^{th} root, etc. When a radical sign carries no index, the index is implicitly understood to be 2, so in that case, we are dealing with the square root.

In NUBS, the index of a radical is attached to the radical sign in the form of a superscript. This differs from the method of indexing a radical as provided in the 1972 Nemeth Code.

RULE (regarding indexed radicals)

If a radical is indexed, the index must be attached to the radical sign in the form of a superscript.

EXAMPLES (regarding indexed radicals)

1 $\sqrt[3]{2}$ ⠼⠉⠩⠼⠃⠜ (the cube root of 2)

2 $3\sqrt[3]{x+y}$ ⠼⠉⠩⠭⠬⠽⠜ (3 times the cube root of *x* plus *y*)

3 $\sqrt[n]{a}$ ⠼⠝⠩⠁⠜ (the n^{th} root of *a*)

4 $\sqrt[m+n]{p+q}$ ⠼⠍⠬⠝⠩⠏⠬⠟⠜ (the *m* plus n^{th} root of *p* plus *q*)

5.7.5 Radicals of Higher Order

Just as in the case of fractions, it may happen that a radicand is itself a radical expression or contains one or more radical expressions. In such situations, we are dealing with *nested radicals.* A *simple radical* is one in which there are no inner radicals. A simple radical is said to be of order 0. For purposes of NUBS only, the order of a radical is not increased if the only radicals that it contains are at the subscript level, the superscript level, or both. An indexed radical can be a simple radical if it contains no inner radicals.

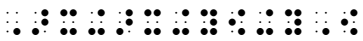
A radical expression that contains inner radicals only one level deep is said to be of order 1. A radical expression that contains inner radicals that are at most two levels deep is said to be of order 2. A radical expression that contains inner radicals that are at most *n* levels deep is said to be of order *n*. In theory, it is possible to have radicals of any order whatever. As a practical matter, we seldom encounter radicals of order higher than 2.

In a manner parallel to the treatment of higher-order fractions, we precede the radical sign and its radical terminator with the complexity indicator, repeated as many times as the order of the radical. Thus, a radical of order 0 requires no complexity indicators; a radical of order 1 requires a single complexity indicator before the radical sign and its terminator; a radical of order 2 requires two complexity indicators before the radical sign and its terminator, etc.

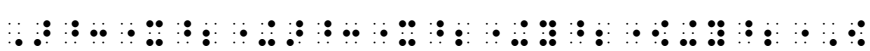
Note that this method of handling radicals of higher order differs from the method provided for this purpose in the 1972 Nemeth Code. This revised method results in the treatment of higher-order fractions and higher-order radicals in a more uniform manner. This revised method also implements the *just-in-time principle,* giving the reader information about the order of the radical with which he must deal just as that radical expression begins.

EXAMPLES (regarding higher-order radicals)

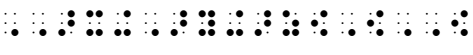
$\sqrt{x+\sqrt{x+y}+y}$

1 

2 $\sqrt[3]{x^2+\sqrt[3]{x^2+y^2}+y^2}$



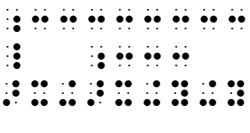
3 $\sqrt{\sqrt[3]{x}}=\sqrt[3]{\sqrt{x}}$ 

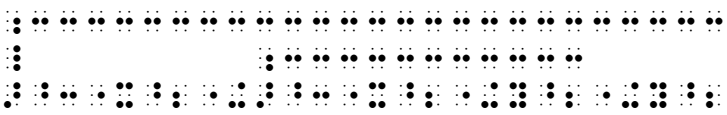
4 $\sqrt{x+\sqrt{y+\sqrt{z}}}$ 

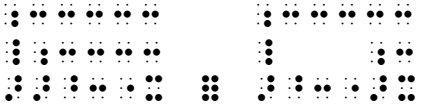
5.7.6 Spatial Representation of Radicals

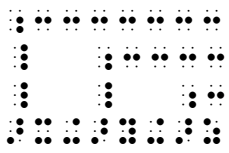
It is possible to represent radicals spatially. A spatial representation is appropriate when the complexity of the radical expression is such that a spatial representation is clearer than a linear representation.

EXAMPLES (showing the spatial representations of the four examples of Section 5.7.5)

1a $\sqrt{x+\sqrt{x+y}+y}$ 

2a $\sqrt[3]{x^2+\sqrt[3]{x^2+y^2}+y^2}$ 

3a $\sqrt{\sqrt[3]{x}}=\sqrt[3]{\sqrt{x}}$ 

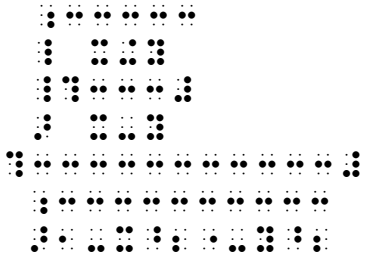
4a $\sqrt{x+\sqrt{y+\sqrt{z}}}$ 

5.7.7 Fractions and Radicals Together in a Spatial Representation

It is possible to have spatial arrangements in which radicals are contained within fractions or in which fractions are contained within radicals. The method is straightforward; nothing more is required than to apply the rules for the representation of spatial fractions and spatial radicals. We present one example as a model.

EXAMPLE (showing the spatial representation of radicals within a fraction)

$$\frac{\sqrt{\frac{x+y}{x-y}}}{\sqrt{1-x^2-y^2}}$$



5.8 Functions and Their Abbreviations

A list of function names and their abbreviations is presented in Section 5.0.7 of the Mathematics Symbol Set. The principal trigonometric function abbreviations are listed separately in Section 5.0.16. These function abbreviations have been accumulated over a period of time from several branches of mathematics. No doubt, others will emerge in due time with more experience. If discovered, they should be included in the list and subjected to the same rules that already apply to the existing list.

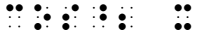
Function abbreviations are printed in the host type. Sometimes, for a special reason, the first letter is printed as an upper-case letter. In such a case, the indicator for an upper-case single letter (dot 6) should be used.

Function abbreviations are notational and contractions should not be used in their representation. Function names, when used in an expository passage rather than in a mathematical expression, may be contracted.

When a subscript, whether numeric or otherwise, is attached to the last letter of a function abbreviation, it should be regarded as applying to the whole function abbreviation, not just to its last letter.

EXAMPLES (regarding function abbreviations)

1 $\sin x$

2 $\cos^2 x$ 

3 $e^{\sin x}$ 

4 arc AOB ⠐⠣⠐⠠⠐⠣⠠⠐⠴

5 $\log_{10}$ x ⠐⠇⠕⠛⠰⠂⠭

6 $\lim_{x \to 0}$ f (x) ⠐⠇⠊⠍⠰⠭⠒⠴ ⠋⠣⠭⠜

5.8.1 Spacing with Function Abbreviations

EXAMPLES (regarding spacing with function abbreviations)

1 sin x + y ⠎⠊⠝⠭ ⠖ ⠽

2 sin $p/3$ ⠎⠊⠝⠏⠸⠌⠼⠉

3 sin 30° cos 45° ⠎⠊⠝⠼⠉⠚⠨ ⠉⠕⠎⠼⠙⠑

4 sin x cos y ⠎⠊⠝⠭ ⠉⠕⠎⠽

5 2sin x +3cos y ⠼⠃⠎⠊⠝⠭ ⠖⠼⠉⠉⠕⠎⠽

6 arc sin x ⠁⠗⠉⠎⠊⠝⠭ (In print, there is a space in the function name.)

7 logsin 60° ⠇⠕⠛⠎⠊⠝⠼⠋⠚⠨ (no space in the function name in print)

5.9 Geometry

5.9.1 Geometric Modifiers

Section 5.0.8.1 of the Mathematics Symbol Set lists the modifiers commonly found in geometry.

In the case of half-lines and line segments as shown in print, a solid dot is used at the end of the modifier if the endpoint is included, and a hollow dot is used if the endpoint is excluded.

EXAMPLES (of geometric modifiers)

1 ⠠⠁⠃ (full-line AB)

2 ⠠⠏⠟ (The half-line PQ excludes its endpoint.)

3 ⠠⠉⠙ (The line segment CD in which C is excluded and D in included.)

5.9.2 Geometric Shapes

Many of the shapes that are the subject matter of plane geometry are represented by little icons rather than by words which identify those shapes. A list of the most common icons

together with their tactile graphics is presented in Section 5.0.8.2 of the Mathematics Symbol Set. They all have the common compound prefix ⠸⠩ .

Note how regular polygons are represented. The root specifies the number of sides (which are all of equal length). This technique applies only to regular polygons; it may be applied to other regular polygons not listed. Thus, a triangle that is not equilateral is represented as ⠸⠩⠣ , whereas an equilateral triangle is represented as ⠸⠩⠒ .

A shape may be shown as hollow, shaded, or filled-in (solid). If it is shown as hollow, nothing more than the shape shown in Section 5.0.8.2 of the Mathematics Symbol Set is required. If the shape is shown as shaded, the root of the shape symbol is preceded by ⠨ . If the shape is shown as solid, the root of the shape symbol is preceded by ⠸ .

In addition, a shape may be shown with a boundary or without a boundary. Of course, a shape that is hollow always has a boundary. If a shaded or a solid shape has a boundary, only the mechanism described in the above paragraph is necessary. But if a shaded or a solid shape has no boundary, the additional prefix ⠈ is placed before the shape indicator ⠸⠩.

EXAMPLES (showing various shapes)

1 ⠸⠩⠸⠩ (a solid rectangle)

2 ⠸⠩⠨⠩ (a shaded square)

3 ⠈⠸⠩⠨⠒ (a shaded circle without a boundary)

When referring to points or vertices of a geometric figure in which upper-case letters are used to name those points or vertices, those letters must be individually capitalized.

EXAMPLES (showing how points or vertices of a geometric figure are represented)

4 Lines AB and CD intersect at P.

⠠⠇⠰⠁⠃ ⠠⠇⠰⠉⠙ ⠍ ⠠⠏⠕⠊⠝⠞ ⠠⠁ ⠠⠏⠕⠊⠝⠞ ⠠⠃ ⠠⠁⠝⠙ ⠠⠏⠕⠊⠝⠞ ⠠⠉ ⠠⠏⠕⠊⠝⠞ ⠠⠙ ⠠⠁⠞ ⠠⠏

5 Δ ABC is a right triangle with hypotenuse BC.

⠸⠩⠁⠃⠉ ⠠⠊⠎ ⠁ ⠗⠊⠛⠓⠞ ⠞⠗⠊⠁⠝⠛⠇⠑ ⠺⠊⠞⠓ ⠓⠽⠏⠕⠞⠑⠝⠥⠎⠑ ⠠⠃⠠⠉
⠠⠁⠝⠙ ⠠⠁⠃ ⠠⠊⠎ ⠁ ⠇⠑⠛

5.10 Grouping Signs (continued)

The basic grouping signs together with their tactile graphics are presented in Section 3.0.9 of the Basic Literary Symbol Set. To those we now add the more technical grouping signs as listed in Section 5.0.9 of the Mathematics Symbol Set.

Most of the time, grouping signs occur in pairs. However, there are times when only the left component of a pair of grouping signs is present without its matching right component, and vice versa. When this occurs, the braille must follow the print.

Vertical bars are mostly used to convey the meaning of "absolute value." In that role, its two identical components enclose the intended expression. However, a vertical bar by itself may be used at the right of an expression and carry a subscript and a superscript to indicate the limits for evaluation. The right bracket by itself is also used for this purpose.

A pair of brackets which enclose only a space is sometimes used to represent the empty set.

The double vertical bar is used with the meaning of "norm."

Here are some examples, all involving grouping signs.

EXAMPLES (showing grouping signs)

2 (i.e.)

3 (s.a.s. = s.a.s.)

4 LCM means *lowest common multiple*.

5 {Wed., Thurs., Fri.}

6 {Mary, Sally, Jean}

7 (seven^2 +1)

8 (light-yr)

9 (*x*-intercept), (*xy*-plane)

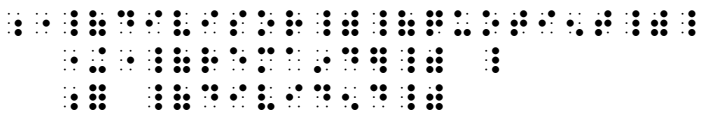
10 (6-inch ruler)
(Note the use of dot 5 to toggle between modes!)

11 (Bar-x)

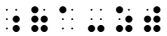
12 (1-to-1)

13 (rate) * (time) = (distance)

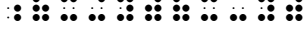
14 (divisor)(quotient) + (remainder) = (dividend)



(Notice the runover indicators at the end of the first two lines.)

15 (a-z) 

16 (VI-IX) 

17 (x+y)(x-y) 

Barred braces and barred brackets are represented as standard braces and brackets but they are preceded by the prefix ⠸ on the left.

Boldface brackets are the standard notation in mathematics for the integer function.

Extended parentheses are used to enclose a binomial coefficient. In print, the components of a binomial coefficient are written one above the other. In NUBS, a binomial coefficient is represented by an extended set of parentheses in which the upper component is followed by the lower component, separated by a space.

EXAMPLE (of binomial coefficient)

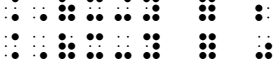
21 $\binom{n!}{(n-k)!}$

In print, extended grouping signs are used to span a determinant, a matrix, or a system of equations. Extended grouping signs should not be used to span a vertically-stacked fraction or to span a tall integral sign which spans a vertically-stacked fraction.

RULE (for the placement of a system spanned by extended grouping signs)

A system spanned by extended grouping signs should be placed so that its top line is abreast of a label, a comparison sign, or an operation sign, if any. If the system is punctuated, the punctuation mark should be placed alongside the top line.

EXAMPLES (of extended grouping signs)

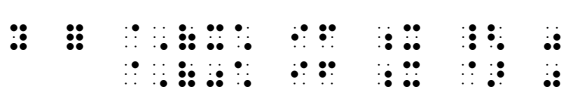
22 $\begin{cases} x+y=2 \\ x-y=0 \end{cases}$ 

(a system of two equations spanned on the left by an extended brace)

23 $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ 

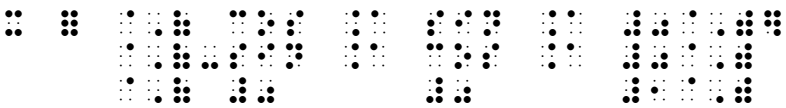
(The evaluation of a 2 x 2 determinant is enclosed by vertical bars. The right side of the equation in print is centered on the extended vertical bar; in NUBS it is abreast of the top line of the extended vertical bar.)

24 $$y = \begin{cases} x, \text{ if } x \le 0 \\ 0, \text{ if } x > 0 \end{cases}$$



(a piecewise-defined equation whose components are spanned on the left by an extended bracket)

25 $$x = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



(A 3 x 3 matrix is spanned on both sides by extended brackets.)

Sometimes a system of equations or a piecewise-defined equation has a comment, narrative or notational, centered so as to apply to the system, but without any extended grouping sign at all. When this happens, NUBS requires an extended transcriber enclosure to span the system so as to associate the comment with the system.

5.11 Logic and Set Theory

Logic and set theory are closely related branches of mathematics and thus share many symbols. Most of those symbols can be classified either as operation signs or as comparison signs, and have been included in their respective categories in the Mathematics Symbol Set together with their tactile graphics. They are governed by the same rules that we have already encountered regarding other operation signs and comparison signs.

The symbols for "meet" and "join" are top-to-bottom symmetric in print; they are likewise top-to-bottom symmetric in NUBS. The same observation holds for the signs for "union" and "intersection."

The prefix  denotes negation. It is placed either before the first character of the principal sign to achieve negation, or it replaces that first character.

The comparison signs in the Logic and Set Theory list have been duplicated in the Signs of Comparison list, and the operation signs in the Logic and Set Theory list have been duplicated in the Operation Signs list. Four entries in the Logic and Set Theory list are neither comparison signs nor operation signs.

5.12 Special Letters (continued)

The special letters recognized by NUBS have been assigned to seven categories. In the full NUBS version,hese are all listed in the Mathematics Symbol Set together with their graphic representation.

5.12.1 Crossed

The crossed b is used in computer displays to denote a blank when it is important to count the actual number of blanks.

The crossed h is used to represent Planck's Constant in the field of quantum mechanics.

5.12.2 Global Operation Signs

The large upper-case boldface sigma is used to represent the sum of a large number of terms. Similarly, the large upper-case boldface pi is used to represent the product of a large number of factors. Other global operation signs that are not letters are entries in the Logic and Set Theory list.

5.12.3 Hebrew

Two Hebrew letters, aleph and beth, are used in the branch of mathematics that deals with transfinite numbers. The 1972 Nemeth Code lists letters of the entire Hebrew alphabet, both in their printed and script forms. However, experience has proved this to be unnecessary.

5.12.4 Miscellaneous

The two letters, partial derivative and Weierstrass p are used in the branch of mathematics that deals with calculus and analysis.

5.12.5 Number Sets

Mathematics employs five letters, each in the same special font, to denote the principal types of numbers: the natural numbers, the integers, the rational numbers, the real numbers, and the complex numbers.

5.12.6 Obsolete Greek

Obsolete Greek letters are used in discussions concerning the ancient Greek numeration system.

5.12.7 Variant Greek

Sometimes two forms of the same Greek letter are used in an expository passage and in the accompanying notational text. When this is the case, the variant form must be distinguished from the standard form, so that NUBS provides the means for doing so.

5.13 Subscripts and Superscripts (continued), and Modifiers

It is characteristic of mathematical notation as well as of notation elsewhere to place some symbols at an elevated or a depressed position relative to the principal line of notation. Symbols that are elevated are called ***superscripts***; symbols that are depressed are called ***subscripts.*** The principal line of notation is called the ***base line.*** Notation, either at an elevated level or at a depressed level, is called an ***index*** (plural, ***indices).***

Subscripts and superscripts have been mentioned in an informal way in Chapter 3 as well as in several earlier sections of this chapter. It is time to give them a coherent analysis.

A superscript is introduced by the indicator ⠔ . It remains in effect until it is cancelled by another level indicator or by a space. Similarly, a subscript is introduced by the indicator ⠢ . It remains in effect until it is cancelled by another level indicator or by a space. The indicator ⠰ is used to return to the base line. Notice how suggestive these indicators are of their functions.

EXAMPLES (of simple subscripts and superscripts)

1 x^2 ⠭⠔⠼⠃ (x square)

2 h_e ⠓⠢⠑ (h sub e) Note how this differs from ⠓⠑ (hence).

3 $x^2 + y^2 = 9$ ⠭⠔⠼⠃⠐⠖⠽⠔⠼⠃⠐⠶⠼⠊ (x square plus y square equals nine)

5.13.1 Primes Together with Indices

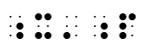
The symbol for "prime" is:

prime ⠄

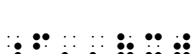
A symbol may carry two or even three primes. In that case, as many dots 3 are supplied in braille as there are primes in print. As with a single prime, no superscript nor base-level indicator is required in braille. If a symbol carries one or more primes, those primes must be written before any subscript or superscript that the symbol may carry.

EXAMPLES (of primes)

1 *x*'+*y*' ⠭⠄⠐⠖⠽⠄

2 x'_{p} 

3 x'^{2} 

4 f " (x) 

5.13.2 Simultaneous and Non-Simultaneous Indices

If a symbol carries both a subscript and a superscript, one below the other, the indices are said to be ***simultaneous,*** and the subscript must be indicated first.

EXAMPLE (of simultaneous indices)

1 $x^{2}_{a} + x^{2}_{b}$ 

(In NUBS, all level indicators are relative to the base line. Thus, in this example, the first 2 is not a superscript to the ***a***, nor is the second 2 a superscript to the ***b***; in both cases the 2's are superscripts to *x*, the symbol at the base line.)

Sometimes, when a symbol carries both a subscript and a superscript, one of them is displaced to the right relative to the other. When one of these is displaced to the right of the other, they are said to be ***non-simultaneous.*** If the symbol on the base line carries one or more primes, causing the superscript to be displaced to the right to make room for the primes, this is not regarded as a true displacement.

In NUBS, when one index is displaced further to the right than the other, and both are indices to the same symbol at the base line, the index closer to the base-line symbol is transcribed first, preceded by the appropriate level indicator. Then comes the base-level indicator. Then comes the other index preceded by its appropriate level indicator.

EXAMPLES (of non-simultaneous indices)

2 $x^{\frac{f}{g}}_{L}$ 

3 $x_{L}^{\frac{f}{g}}$

In Example 2, the superscript is closer to *x* than the subscript; in Example 3, the subscript is closer to *x* than the superscript. The base-level indicator between the two indices tells the reader that they are not simultaneous. The order in which the reader encounters the indices tells him which comes first and which comes second.

5.13.3 Left Indices (Omitted in condensed version)

5.13.4 Numeric Subscripts

Numeric subscripts are by far the most commonly encountered subscripts in mathematics. Therefore, NUBS has devised a shortcut for their representation. The rules are as follows:

RULES (for numeric subscripts)

a *The rules for numeric subscripts apply only to letters.* The letters may be lower case or upper case; they may be in any font, and they may be from any alphabet, but they must be at the base level.

b *The numeric subscript may contain a decimal point, commas, or any combination of these.*

c *When a number follows a letter, it is assumed to be a subscript to that letter.* A subscript indicator is not required and no level indicator after the number is required to return to the base line.

d *When a number that follows a letter is not a subscript to that letter but is at the same level as the letter, numeric indicator must be inserted before the number.*

EXAMPLES (of numeric subscripts)

1 x_1+x_2 (*x* sub 1 plus *x* sub 2)

2 $a_{11}x_1+a_{12}x_2+ \ \ldots \ +a_{1n}x_n = b_1$

(The first equation in a system of linear equations.)

3 $p_1^{a_1}p_2^{a_2} \cdots p_n^{a_n}$

(decomposition of a composite number into its constituent prime factors)

4 Vitamin B12 (The 12 is not a subscript.)

5.13.5 Higher-Order Indices

When a symbol is already a subscript or a superscript, there is nothing to preclude the possibility that such a symbol has a subscript or a superscript of its own. And, in turn, there is nothing to preclude these second-level subscripts or superscripts from having third-level subscripts or superscripts. While there is no theoretical limit to the number of subscript or superscript levels, the practical limit is two, or at most three levels.

In NUBS, a higher-level subscript or superscript is indicated by a sequence of level indicators which leads the reader on a path beginning at the base level and ending at the higher-level symbol. Thus, ⠘⠘ takes the reader from the base level to the "up-up" level; ⠰⠘ takes him from the base level to the "down-up" level; ⠘⠰⠘ takes him from the base level to the "up-down-up" level, etc.

Once a level has been indicated, it remains in effect until it is terminated by another level indicator or by a space.

EXAMPLES (of higher-order indices)

1	$\int e^{-x^{2}}\mathrm{dx}$	⠰⠮⠑⠘⠤⠭⠘⠘⠆⠐⠙⠭
2	$e^{x^{2}+1}$	⠰⠑⠘⠭⠘⠘⠆⠘⠬⠂
3	$e^{x^{2}} + 1$	⠰⠑⠘⠭⠘⠘⠆⠐⠬⠂

In Example 2, the 1 is at the "up" level, abreast of the *x*. In Example 3, the 1 is at the base level, abreast of the *e* .

5.13.6 Overscripts and Underscripts

In addition to subscripts and superscripts, mathematical notation permits stacked notation. In notation of this kind, the principal expression is modified by subsidiary expressions which are written below the principal expression, above the principal expression, or both. Subsidiary expressions that are written below the principal expression are called *underscripts;* subsidiary expressions written above the principal expression are called *overscripts.* The principal expression may contain more than one underscript, more than one overscript, or more than one of both. These are written at successively more remote levels relative to the principal expression.

In NUBS, a first-level underscript is indicated by:

⠰⠯

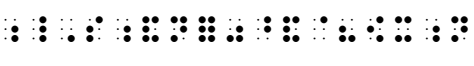
A first-level overscript is indicated by:

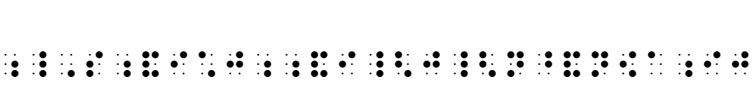
⠘⠯

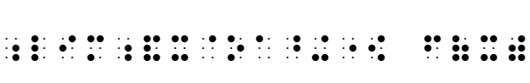
The termination indicator ⠪ . must be used to indicate that we have come to the end of the stack. Note that this termination indicator is the same as is used for terminating a radical.

EXAMPLES (of underscripts and overscripts)

1 $\lim_{x \to 0} f(x)$ 

2 $\sum_{n=0}^{\infty} x_n$ 

3 $\sum_{\substack{i,j \\ i \le j \le n}}^{n} a_{ij}$ 

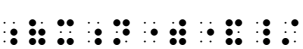
4 $\lim_{x \to a^+} f(x)$ 

NUBS also provides an *overstrike* mechanism by which one symbol overstrikes another. The APL computer language uses this mechanism extensively. In mainstream notation, a slash is made to overstrike another symbol or expression to indicate that that symbol or expression is cancelled. The overstrike is indicated by:

overstrike indicator 

EXAMPLES (of the overstrike mechanism)

5 $\not\in$  (The equals sign overstrikes the inclusion sign.)

6 $\not{(x_n)}$ 

(The slash overstrikes the expression (x_n) to indicate that it is cancelled.)

5.13.7 Modifiers

Although subscripts, superscripts, underscripts, and overscripts can properly be thought of as modifiers of the expressions to which they are attached, there is a certain class of symbols which we will directly designate as *modifiers.* This class of modifiers contains lines, dots, arrows, and accent marks. These modifiers can be found above the expression which they affect, or below the expression which they affect.

If the modifier is an overscript, the expression which is modified is preceded by  ; if the modifier is an underscript, the expression which is modified is preceded by  . (These are the "directly-over" and the "directly-under" indicators of the 1972 Nemeth Code.)

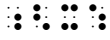
The expression to be modified is then followed by the specific modifier. As is the case with normal underscripts and overscripts, a modified expression is a stack. In the case of modifiers, however, a termination indicator is not required; the stack ends with the last modifier. The

following is a list of modifiers recognized by NUBS. If a symbol is not on this list, it must be treated in the manner of an underscript or an overscript as described in the preceding section.

Modifiers

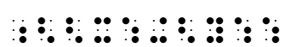
acute accent		left arrow	
asterisk		macron	
breve		right arrow	
circumflex accent		ring	
diaeresis		tilde	
dot		two-way arrow	
grave accent		umlaut	
hat			

EXAMPLES (of modifiers)

1 $\overline{x}$  (bar over *x*)

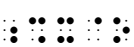
2 $\dot{x}$  (dot over *x*)

3 $3.\overline{142857}$  (a repeating decimal)

4. $\overline{\overline{x}+\overline{y}}$ 

(the conjugate of, the conjugate of *x* plus the conjugate of *y*.)

5 $\hat{xy}$  (hat over *xy*)

6 $\underset{\rightarrow}{x}$  (right arrow under *x*.)

5.14 Trigonometry

Trigonometry is characterized by the generous use of function abbreviations. It is customary for these function abbreviations to be printed in the host type. The function abbreviation for cotangent has two commonly used forms, namely, cot and ctn. There are other function abbreviations which are not specific to trigonometry, and these have been listed separately in Section 5.0.7 of the Mathematics Symbol Set. Other than these function abbreviations, the notation in trigonometry is the same as that used in other branches of mathematics.

Some now obsolete trigonometric function abbreviations are included in the list of Function Abbreviations.

This index cover the entire NUBS document, Chapters 0 through 8.

This index is not included in the condensed version since the page numbers in print and braille shown would not match the pages in the shorter version.

Each entry in the index includes the section number and the print page number. In the braille version, this is followed by an additional notation to provided the braille volume and braille page number. For example, a reference of:

ANGLE BRACKETS 3.2.3 49 (4.5)

indicated that angle brackets are discussed in Section 3.2.3, on print page 49, and in braille volume 4 on braille page 5. Since indexed items might not occur at the beginning of the referenced section, the print and braille pages listed may not included the start of that section.

END OF THE CONDENSED VERSION OF NUBS